On the Importance of Mastering Solution Methods of Elementary Number Theory Problems

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Number theory problems are widely represented at various mathematical Olympiads. The themes of problems comprise questions on the divisibility of integers, factorization, parity, co-prime numbers, modular arithmetic, Diophantine equations, etc. These problems are important not only because they investigate different properties of integers, but also because a variety of methods can be applied in solving them, including algebraic transforms, combinatorial calculations, and logical statements. In solving problems of various degrees of difficulty, general methods of reasoning may be applied, such as mathematical induction, the method of invariants, the method of the extreme element, and the mean value method.

One of special cases of the mean value method is the Dirichlet's box principle (DP) (Andžāns, 2005). It can be applied in solving problems on the string of number's digits, and problems on the powers of integers. It can be applied in solving various problems on finite sets of integers, where the properties of a subset must be considered, e.g. where if must be proved that a particular subset will contain a certain group of numbers. With application of DP and the fundamental theorem of arithmetic, properties of the sequence of consecutive numbers can be investigated. Many problems of modular arithmetic can be solved by applying DP or such well-known results as Fermath's - Euler's theorem, Wilson's theorem (Сендеров,1999), or Chinese remainder theorem, which are all in essence special cases of mean value method. By applying the infinity descent method, one can prove the divisibility of numbers or study particular properties of the division process or solve certain Diophant equations (PEN, 2008).

It is important to know the elementary number theory results and problem solving methods to apply them in solutions of various combinatorial problems. Those include problems on finite sets, where numerical estimations must be made to detect the properties of the elements of subsets. Such estimations can be made by applying DP or the well-known Cauchy theorem. In combinatorial geometry problems on figures defined in the orthogonal lattice or in problems on Ramsey's figures, the given objects can be classified by applying modular arithmetic. The grouping of objects in equivalence classes according to a given modulus allows comparing the cardinality of those classes by DP. The application of this method is useful in solving graph theory problems on graph routing, combinatorial problems on placing of numbers in tables, cryptographic problems, and others. By applying the infinite descent theory, the continuity of an algorithmic process can be tested.

School curricula include relatively few elementary number theory problems with proof. Nevertheless, the solution methods of those problems form the base of many important mathematical results. Therefore, it is significant that students learn the basics of elementary number theory already in primary school and develop this knowledge gradually, not only in order to participate in mathematical Olympiads, but also to gain a deeper understanding on the interrelation of various branches of mathematics.

References

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