School-elementary proofs of irrationality

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Proofs of irrationality were collected in I. Niven's (1956) book, but only several of them can be called elementary. As a rule, almost the same nonelementary proofs are presented on the internet. However, much time has elapsed since the appearance of the book and many proofs have been successfully simplified and made accessible even for a pupil. By the way, the fact that the proofs of this kind are of importance in the system of school knowledge was understood long ago: in many textbooks there are even respective chapters.
Let us discuss a number of elementary proofs.

The proofs of $\sqrt{2}$ irrationality are well known. That is a nice example how the methods of infinite descent, contradiction, and that of decomposition into prime factors are applied. At present the geometric proof of incommensurability of the square side and diagonal is a little forgotten, but it is namely this fact that compelled Euclid to create the theory of proportions os segments, which made the basis of his geometric system. It is not so far from the irrationality of $\sqrt{2}$ to the irrationality of all radicals.

The proof of $\log_2 3$ (and of other logarithms) irrationality is also based on the decomposition into prime factors.

The proofs of irrationality of $\cos 17^\circ$ (and the values of other trigonometric functions at the points $m\pi / n$) offer unlimited possibilities for inventiveness by operating with trigonometric formulas. The brief general proof (hardly of several lines), is poorly known (Mačys 2008).

The proof for $\cos 1$ (and for some other values at the points *m/n*) demonstrates that we can do even without the Taylor series (see op. cit. and references).

School-elementary proofs of e and π irrationality are quite new and little known (Mačys 2009). The desire of inquisitive students to know such proofs is quite natural. At present there emerged a good possibility for that.

References

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