Individual Assignments, Tests, Examination Problems: Insight into First Year Students' Solutions

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University teachers often complain that pupils leave school equipped with techniques at best but with little idea of fundamental concepts of mathematics and of what mathematics is really about and also that they show poor appreciation of the need for logical precision. The question: *why many students obtaining the highest level in the centralized examination in mathematics* (*level A*) *show a weak progress in mathematics courses lectured in the first semester at universities* will be discussed.

The focus here is on the problems involving a limit of a sequence or a function, in particular students' failures, howlers, fallacies, and didactic errors. For present purposes banal mistakes, blunders are of little significance and have no interest. "The term *howler* is used to denote an error which leads *innocently* to a correct answer, a result. By contrast, the *fallacy* leads by *guile* to a wrong but plausible conclusion." [1]. It seems to be the trend in many universities for students to have a difficulty performing even the most basic mathematical calculations.

A few examples

Calculate the compositions: $(f\circ f)(-2), (f\circ f)(x), (f\circ f)(|x|)$ if $f(x)=\sqrt{16-(x+5)^2}-5$.

Prove or disprove the equality $\lim(a_n - b_nc_n) = \lim(a_n - b_n)$ if $\lim c_n = 1$.

The typical reasoning of students is as follows:

$$\lim(a_n-b_nc_n)=\lim a_n-\lim b_nc_n=\lim a_n-\lim b_n\cdot\lim c_n=\lim a_n-\lim b_n\cdot 1=\lim(a_n-b_n).$$

However, the equality must be disproved.

Find a limit:

$$\lim_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}};\quad \lim_{x\to 0}\frac{|x|(1-\cos x)}{|x|+\sin x};\quad \lim_{x\to\infty}\left(\frac{x}{e}-x\left(1+\frac{1}{x}\right)^{-x}\right).$$

Solutions of tests, individual tasks, and examination problems made by freshmen frequently are unreasonable. Yet the standard problems, e. g. the first example of the mentioned ones which is taken from [2], were solved by many students in a long, laborious, inefficient way.

Find the integral part of the number

$$\sqrt{6+\sqrt{6+\sqrt{6+\ldots+\sqrt{6}}}}+\sqrt[3]{6+\sqrt[3]{6+\sqrt[3]{6+\cdots+\sqrt[3]{6}}}}\ .$$

Noticing that this example, see [3], is for the secondary school seniors, we can conclude that 13 years ago the requirements for the mathematical background of pupils were higher than they are nowadays/at present.

Are your students able to solve the mentioned examples?

References

[1] Maxwell E. A., *Fallacies in Mathematics*, Cambridge University Press, 1959. http://www.scribd.com/doc/19346327/Fallacies-in-Mathematics

[2] Кузнецов Л. А., *Сборник заданий по высшей математике*, Москва, Высшая школа, 1983, 176 с. [http://www.reshebnik.ru/solutions/6/12/]

[3] Математика в школе, 1987, №1.