

On the Necessity of Verification of Sufficient Conditions

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The process of teaching mathematics involves not only teaching mathematical knowledge, but also teaching the correct use of mathematical concepts and results. The correct use of formulas and theorems is not possible without verification of all assumptions under which the results hold, i.e. the sufficient conditions.

When students apply a formula or a theorem, they often concentrate on methods and techniques of calculations and pay little attention to properties of functions, the validity of conditions, etc. Students often do not bother themselves to check that the sufficient conditions are satisfied: they do not recognize the significance of such conditions or simply do not think about them at all. Our experience in teaching Mathematical Analysis shows that in order to teach students to verify sufficient conditions it is necessary for a lecturer:

- 1) to formulate all assumptions, under which the result is true;
- 2) to emphasize the role of each one of the sufficient conditions;
- 3) to consider examples of typical mistakes when the result is used incorrectly.

In case we follow this practice, students get accustomed to pay attention to every detail of a statement. For the deeper understanding of the result it is helpful for students to have a list of certain type examples, which satisfy all the assumptions except for one in the considered theorem without the conclusion being valid. Each example illustrates why the theorem will be invalid if the corresponding assumption is violated. In many cases a simple sketch of a graph is enough to create such an example. But there are some cases when examples illustrating the significance of sufficient conditions are not so elementary. A set of corresponding examples is considered in the report. These examples are mainly taken from the course of Mathematical Analysis.

We present also some examples of solutions in which students often make mistakes and obtain wrong results because of ignoring the sufficient conditions. Undoubtedly the analysis of mistakes is a good teaching tool in mathematics. There are different ways of using such examples: giving students a mix of correct and incorrect calculations, making a deliberate mistake in calculations and asking students to spot the error, etc.

The suggested practice is one of the numerous pedagogical strategies in teaching mathematics. In our opinion such practice helps students to understand mathematical concepts better and to prevent mistakes in future.