On the Connection Between Double and Simple Zeros of Solutions of the Third-Order Linear Differential Equations

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Summary. Zero properties of solutions of a third-order linear ordinary differential equation are discussed. Adjoint equation and equation which has similar properties concerning distribution of zeros are considered. More general equation with the same properties is established. Illustrative examples and figures are provided.

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1 Introduction

The main purpose of this paper is to consider zero properties for solutions of the third-order linear differential equation of the form

$$x''' + p(t)x'' + q(t)x' + r(t)x = 0, \tag{1}$$

where $p(t)$, $q(t)$ and $r(t)$ are real-valued, continuous functions defined on an interval I unless explicitly stated otherwise.

This work was stimulated by the investigations of third-order equations by N.V. Azbelev and Z.B. Caljuk [1]. Results concerning distribution of zeros were obtained by M. Hanan [2], W. J. Kim [3].

In section 2 we consider some basic concepts and results which are used in later sections.

In section 3 we consider the connection between double and simple zeros of the equation (1) and its adjoint.

In section 4 we investigate the equation (we call it by Kim's equation) which has similar properties with adjoint equation concerning distribution of zeros.

In section 5 we establish more general equation with the same properties. Adjoint equation and Kim's equation are the special cases of this equation.

2 Preliminaries

Lemma 2.1 *If $x(t)$ is a solution of the equation* (1) *and if there exist $t_0 \in I$ such that $x(t_0) = x'(t_0) = x''(t_0) = 0$, then $x(t)$ is trivial solution on I $(x(t) \equiv 0)$.*

Corollary 2.1 *If $t_0 \in I$ is a zero of the nontrivial solution of the equation* (1)*, then its multiplicity can be equal only to 1 or 2.*

Lemma 2.2 *If $x_1(t)$ and $x_2(t)$ are two linearly independent solutions of* (1) *which vanish at $t = a$ then any other solution vanishing at this point can be written in the form*

$$x(t) = c_1x_1(t) + c_2x_2(t).$$

Proof. If $x_1(t)$, $x_2(t)$, $x_3(t)$ are linearly independent solutions of (1), then the general solution of (1) is

$$x(t) = c_1x_1(t) + c_2x_2(t) + c_3x_3(t),$$

where c_1, c_2, c_3 denote arbitrary constants.
Assume that

$$x_1(a) = x_2(a) = 0.$$

If $x(a) = 0$, then

$$x(t) = c_1x_1(a) + c_2x_2(a) + c_3x_3(a) = 0.$$

Hence $c_3 = 0$ and $x(t) = c_1x_1(t) + c_2x_2(t)$.□

Lemma 2.3 *Suppose that $x_1(t)$ and $x_2(t)$ are two linearly independent solutions of* (1) *such that $x_1(a) = x_2(a) = 0$, and $x(t)$ is a solution of* (1) *which vanishes at $t = a$ $(x(a) = 0)$. A necessary and sufficient condition for $t = \tau$ to be a double zero for $x(t)$ $(x(\tau) = x'(\tau) = 0)$ is that the determinant*

$$\begin{vmatrix} x_1(\tau) & x_2(\tau) \\ x_1'(\tau) & x_2'(\tau) \end{vmatrix}$$

is equal to zero.

Proof. Necessity. Since $x(a) = 0$ then $x(t) = c_1x_1(t) + c_2x_2(t)$.
If $t = \tau$ is a double zero for $x(t)$ then

$$\begin{cases} c_1x_1(\tau) + c_2x_2(\tau) = 0, \\ c_1x_1'(\tau) + c_2x_2'(\tau) = 0. \end{cases}$$

Since $c_1^2 + c_2^2 > 0$ then

$$\begin{vmatrix} x_1(\tau) & x_2(\tau) \\ x_1'(\tau) & x_2'(\tau) \end{vmatrix} = 0.$$

Sufficiency. Since $x(a) = 0$ then

$$x(t) = c_1x_1(t) + c_2x_2(t)$$

and

$$x'(t) = c_1 x_1'(t) + c_2 x_2'(t).$$

Since

$$\begin{vmatrix} x_1(\tau) & x_2(\tau) \\ x_1'(\tau) & x_2'(\tau) \end{vmatrix} = 0$$

then

$$\begin{cases} x_1(\tau) = c x_2(\tau), \\ x_1'(\tau) = c x_2'(\tau). \end{cases}$$

Therefore, we have

$$\begin{cases} x_1(\tau) - c x_2(\tau) = 0, \\ x_1'(\tau) - c x_2'(\tau) = 0. \end{cases}$$

If we denote $c = -\dfrac{c_2}{c_1}$ we get

$$\begin{cases} x_1(\tau) + \dfrac{c_2}{c_1} x_2(\tau) = 0, \\ x_1'(\tau) + \dfrac{c_2}{c_1} x_2'(\tau) = 0, \end{cases}$$

or

$$\begin{cases} c_1 x_1(\tau) + c_2 x_2(\tau) = 0, \\ c_1 x_1'(\tau) + c_2 x_2'(\tau) = 0. \end{cases}$$

Hence $x(\tau) = 0$ and $x'(\tau) = 0$.
Therefore $x(t)$ has a double zero at $t = \tau$.□

Lemma 2.4 *If $x(t)$ and $y(t)$ are two solutions of the equation (1) and $x(t_0) = x'(t_0) = y(t_0) = y'(t_0) = 0$, $t_0 \in I$ (both solutions have a double zero at the same point), then $x(t)$ and $y(t)$ are linearly dependent on I.*

Lemma 2.5 *If $x(t)$ and $y(t)$ are two solutions of the equation (1) and $x(t_1) = y(t_1) = x(t_2) = y(t_2) = 0$, $t_1, t_2 \in I$ (solutions have two common zeros at two distinct points), then $x(t)$ and $y(t)$ are linearly dependent on I.*

3 Adjoint equation

In the study of equation (1), its adjoint

$$z''' - (p(t)z)'' + (q(t)z)' - r(t)z = 0$$

or

$$z''' - pz'' + (q - 2p')z' + (q' - p'' - r)z = 0 \tag{2}$$

plays an important role.
The self-adjoint form of the third-order equation is [2]

$$z''' + pz' + \frac{1}{2}p'y = 0.$$

It is known [1] that if (1) has a nontrivial solution with three zeros on I, then there is a nontrivial solution x of (1) which satisfies at least one set of the boundary conditions

$$x(a) = x'(a) = x(b) = 0, \tag{3}$$

$$x(a) = x(b) = x'(b) = 0, \tag{4}$$

where $a,\ b \in I$.
Furthermore, if (1) with $p \in C^2$, $q \in C^1$, $r \in C$ has a nontrivial solution satisfying condition (3) (respective (4)), then its adjoin equation (2) has a nontrivial solution which satisfies condition (4) (respective(3)).

It is easy to show [1], that if $x_1(t)$, $x_2(t)$ and $x_3(t)$ are three linearly independent solutions of the equation (1) and

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ x_1' & x_2' & x_3' \\ x_1'' & x_2'' & x_3'' \end{vmatrix} = W \neq 0, \tag{5}$$

then the functions

$$z_1(t) = \frac{1}{W}\begin{vmatrix} x_1 & x_2 \\ x_1' & x_2' \end{vmatrix},\ z_2(t) = \frac{1}{W}\begin{vmatrix} x_1 & x_3 \\ x_1' & x_3' \end{vmatrix},\ z_3(t) = \frac{1}{W}\begin{vmatrix} x_2 & x_3 \\ x_2' & x_3' \end{vmatrix} \tag{6}$$

are three linearly independent solutions of the equation (2).

Example 3.1 *Consider equation*

$$x''' + x'' + x' + x = 0. \tag{7}$$

e^{-t}, $\cos t$, $\sin t$ *is the fundamental set of solutions of the equation* (7). *Equation*

$$x''' - x'' + x' - x = 0 \tag{8}$$

is adjoint to (7). *The fundamental set of solutions of the adjoint equation* (8) *is* e^t, $\cos t$, $\sin t$. *Equation* (7) *has a solution which vanishes at* $t = 0$ *and has a double zero at* $t = \tau$ ($\tau \approx -3.95$).
Hence, the equation (8) *has a solution which vanishes at* $t = \tau$ *and has a double zero at* $t = 0$.

4 Kim's equation

Lemma 4.1 *Assume that* $x_1(t)$ *and* $x_2(t)$ *are two linearly independent solutions of* (1). *If*

$$W_{01} = \begin{vmatrix} x_1 & x_2 \\ x_1' & x_2' \end{vmatrix},\quad W_{02} = \begin{vmatrix} x_1 & x_2 \\ x_1'' & x_2'' \end{vmatrix},\quad W_{12} = \begin{vmatrix} x_1' & x_2' \\ x_1'' & x_2'' \end{vmatrix},$$

then the relations

$$\begin{aligned} W_{01}' &= W_{02}, \\ W_{02}' &= W_{12} - pW_{02} - qW_{01}, \\ W_{12}' &= -pW_{12} + rW_{01} \end{aligned}$$

are valid, where p, q, r *are the coefficients from the equation* (1).

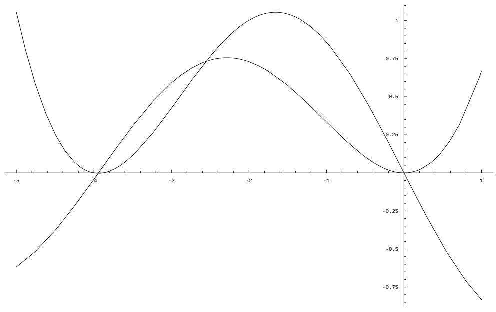

Figure 3.1 Solutions of the problems (7), $x(0) = x(\tau) = x'(\tau) = 0$, and (8), $x(0) = x'(0) = x(\tau) = 0$.

This statement was formulated in [3] and given without proof. We give the proof here for the reader's convenience.
Proof.

$$W_{01}' = (x_1x_2' - x_1'x_2)' = x_1'x_2' + x_1x_2'' - x_1''x_2 - x_1'x_2' = W_{02}.$$

$$\begin{aligned} W_{02}' = (x_1x_2'' - x_1''x_2)' = x_1'x_2'' + x_1x_2''' - x_1'''x_2 - x_1''x_2' = (x_1'x_2'' - x_1''x_2') + x_1x_2''' - x_1'''x_2 = \\ = W_{12} - px_1x_2'' - qx_1x_2' - rx_1x_2 + px_1''x_2 + qx_1'x_2 + rx_1x_2 = \\ = W_{12} - p(x_1x_2'' - x_1''x_2) - q(x_1x_2' - x_1'x_2) = W_{12} - pW_{02} - qW_{01}. \end{aligned}$$

$$\begin{aligned} W_{12}' = (x_1'x_2'' - x_1''x_2')' = x_1''x_2'' + x_1'x_2''' - x_1'''x_2' - x_1''x_2'' = x_1'x_2''' - x_1'''x_2' = \\ = -px_1'x_2'' - qx_1'x_2' - rx_1'x_2 + px_1''x_2' + qx_1'x_2' + rx_1x_2' = -p(x_1'x_2'' - x_1''x_2') + r(x_1x_2' - x_1'x_2) = \\ = -pW_{12} + rW_{01}.\square \end{aligned}$$

Consider the third-order linear differential equation of the form

$$y''' + 2p(t) \cdot y'' + \Big(p'(t) + q(t) + \big(p(t)\big)^2\Big) \cdot y' + \Big(q'(t) + p(t)q(t) - r(t)\Big) \cdot y = 0, \tag{9}$$

where $p(t)$, $q(t) \in C_I^1$ and $r(t) \in C_I$. We will call the equation (9) by Kim's equation.

Lemma 4.2 *If $x_1(t)$ and $x_2(t)$ are two linearly independent solutions of* (1)*, then the function $y(t) = x_1(t)x_2'(t) - x_1'(t)x_2(t)$ is the solution of the equation* (9)*.*

Proof. Denoted $W_{01} = y_1$, $W_{02} = y_2$, $W_{12} = y_3$, we get normal system

$$\begin{cases} y_1' = y_2, \\ y_2' = y_3 - py_2 - qy_1, \\ y_3' = -py_3 + ry_1. \end{cases} \tag{10}$$

Obviously, that $(y_1,\ y_2,\ y_3)$ is a solution of the system (10).
From the second equation

$$\begin{aligned} y_3 &= y_2' + py_2 + qy_1, \\ y_3' &= y_2'' + p'y_2 + py_2' + q'y_1 + qy_1'. \end{aligned}$$

Hence we get

$$\begin{cases} y_1' = y_2, \\ y_2'' + p'y_2 + py_2' + q'y_1 + qy_1' + py_2' + p^2y_2 + pqy_1 - ry_1 = 0. \end{cases}$$

And denoted $y_2' = y_4$ we obtain

$$\begin{cases} y_1' = y_2, \\ y_2' = y_4, \\ y_4' = -2py_4 - (p' + q + p^2)y_2 - (q' + pq - r)y_1. \end{cases} \tag{11}$$

Hence, the system (11) is equivalent to the linear equation (9).
Since $y(t) = x_1(t)x_2'(t) - x_1'(t)x_2(t)$ is the first component of a solution to the system (10) and to the system (11), then $y(t)$ is the solution of the equation (9). □

Lemma 4.3 *If $x_1(t)$, $x_2(t)$ and $x_3(t)$ are three linearly independent solutions of the equation* (1)*, then the functions*

$$y_1(t) = \begin{vmatrix} x_1 & x_2 \\ x_1' & x_2' \end{vmatrix},\ y_2(t) = \begin{vmatrix} x_1 & x_3 \\ x_1' & x_3' \end{vmatrix},\ y_3(t) = \begin{vmatrix} x_2 & x_3 \\ x_2' & x_3' \end{vmatrix}$$

are three linearly independent solutions of the equation (9) *and*

$$\begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = W^2,$$

where W is given in (5)*.*

Proof. Let us denote

$$W_{01}^{ij} = \begin{vmatrix} x_i & x_j \\ x_i' & x_j' \end{vmatrix},\ W_{02}^{ij} = \begin{vmatrix} x_i & x_j \\ x_i'' & x_j'' \end{vmatrix},\ W_{01}^{ij} = \begin{vmatrix} x_i' & x_j' \\ x_i'' & x_j'' \end{vmatrix},$$

$i,\ j = 1,\ 2,\ 3,\ 1 \le i < j \le 3.$
Hence

$$\omega = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ (W_{01}^{12})' & (W_{01}^{13})' & (W_{01}^{23})' \\ (W_{01}^{12})'' & (W_{01}^{13})'' & (W_{01}^{23})'' \end{vmatrix} =$$

$$= \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ W_{02}^{12} & W_{02}^{13} & W_{02}^{23} \\ W_{12}^{12} - pW_{02}^{12} - qW_{01}^{12} & W_{12}^{13} - pW_{02}^{13} - qW_{01}^{13} & W_{12}^{23} - pW_{02}^{23} - qW_{01}^{23} \end{vmatrix} =$$

$$= \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ W_{02}^{12} & W_{02}^{13} & W_{02}^{23} \\ W_{12}^{12} & W_{12}^{13} & W_{12}^{23} \end{vmatrix} + \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ W_{02}^{12} & W_{02}^{13} & W_{02}^{23} \\ -pW_{02}^{12} & -pW_{02}^{13} & -pW_{02}^{23} \end{vmatrix} + \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ W_{02}^{12} & W_{02}^{13} & W_{02}^{23} \\ -qW_{01}^{12} & -qW_{01}^{13} & -qW_{01}^{23} \end{vmatrix} =$$

$$= \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ W_{02}^{12} & W_{02}^{13} & W_{02}^{23} \\ W_{12}^{12} & W_{12}^{13} & W_{12}^{23} \end{vmatrix}.$$

Setting

$$A = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_1' & x_2' & x_3' \\ x_1'' & x_2'' & x_3'' \end{pmatrix},$$

$$A^{-1} = \frac{1}{W} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \frac{1}{W} \begin{pmatrix} W_{12}^{23} & -W_{02}^{23} & W_{01}^{23} \\ -W_{12}^{13} & W_{02}^{13} & -W_{01}^{13} \\ W_{12}^{12} & -W_{02}^{12} & W_{01}^{12} \end{pmatrix} = \frac{1}{W} \cdot B.$$

Since $A \cdot A^{-1} = E$ then $A \cdot \dfrac{B}{W} = E$ or $A \cdot B = WE$.

Since $\det(A \cdot B) = \det(WE) = W^3$ and $\det(A \cdot B) = \det A \cdot \det B$ then $W^3 = W \cdot \det B$ or

$$\det B = W^2.$$

$$\det B = \begin{vmatrix} W_{12}^{23} & -W_{02}^{23} & W_{01}^{23} \\ -W_{12}^{13} & W_{02}^{13} & -W_{01}^{13} \\ W_{12}^{12} & -W_{02}^{12} & W_{01}^{12} \end{vmatrix} = \begin{vmatrix} W_{12}^{23} & -W_{12}^{13} & W_{12}^{12} \\ -W_{02}^{23} & W_{02}^{13} & -W_{02}^{12} \\ W_{01}^{23} & -W_{01}^{13} & W_{01}^{12} \end{vmatrix} = - \begin{vmatrix} W_{01}^{23} & -W_{01}^{13} & W_{01}^{12} \\ -W_{02}^{23} & W_{02}^{13} & -W_{02}^{12} \\ W_{12}^{23} & -W_{12}^{13} & W_{12}^{12} \end{vmatrix} =$$

$$= \begin{vmatrix} W_{01}^{12} & -W_{01}^{13} & W_{01}^{23} \\ -W_{02}^{12} & W_{02}^{13} & -W_{02}^{23} \\ W_{12}^{12} & -W_{12}^{13} & W_{12}^{23} \end{vmatrix} = \begin{vmatrix} W_{01}^{12} & W_{01}^{13} & W_{01}^{23} \\ W_{02}^{12} & W_{02}^{13} & W_{02}^{23} \\ W_{12}^{12} & W_{12}^{13} & W_{12}^{23} \end{vmatrix} = \omega.$$

$$\omega = \det B = W^2.$$

$$\omega = W^2. \ \Box$$

Theorem 4.1 *If $x(t)$ is a nontrivial solution of the equation* (1) *which vanishes at $t = a$ ($x(a) = 0$), has a double zero at $t = \tau$ ($x(\tau) = x'(\tau) = 0$), then the solution $y(t)$ of equation* (9)*, which has a double zero at $t = a$ ($y(a) = y'(a) = 0$) vanishes at $t = \tau$ ($y(\tau) = 0$).*

Proof. Let $x(t)$ be a solution of the equation (1), which vanishes at $t = a$. Then (by the lemma 2.2) this solution can be written in the form $x(t) = c_1x_1(t) + c_2x_2(t)$, where $x_1(t)$ and $x_2(t)$ are two linearly independent solutions of (1) which vanish at $t = a$, and c_1, c_2 denote arbitrary constants.

Since at $t = \tau$ $x(\tau) = x'(\tau) = 0$, then (by the lemma 2.3) $\begin{vmatrix} x_1(\tau) & x_2(\tau) \\ x_1'(\tau) & x_2'(\tau) \end{vmatrix} = 0.$

By the lemma 4.2 $y(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ x_1'(t) & x_2'(t) \end{vmatrix}$ is the solution of the equation (9),

and $y(a) = y'(a) = 0$.

Since $\begin{vmatrix} x_1(\tau) & x_2(\tau) \\ x_1'(\tau) & x_2'(\tau) \end{vmatrix} = 0$, then $y(\tau) = 0$.

Hence the solution of the equation (9) which has a double zero at $t = a$ vanishes at $t = \tau$.$\Box$

Example 4.1 *Kim's equation for the equation* (7) *is*

$$x''' + 2x'' + 2x' = 0. \tag{12}$$

Equation (12) *is easier than adjoint equation* (8)*. The fundamental set of solutions of the Kim's equation* (12) *is* $1,\ e^{-t}(\sin t - \cos t),\ e^{-t}(\sin t + \cos t)$.

The solution of the equation (12) *which has a double zero at $t = 0$ vanishes at $t = \tau$.*

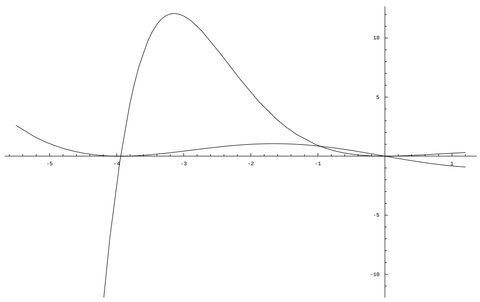

Figure 4.1 Solutions of the problems (7),
$x(0) = x(\tau) = x'(\tau) = 0$, and
(12), $x(0) = x'(0) = x(\tau) = 0$.

5 Generalized equation

Suppose that $f(t) \neq 0 \ \forall t \in I$. It is easy to prove that

$$\begin{vmatrix} fy_1 & fy_2 & fy_3 \\ (fy_1)' & (fy_2)' & (fy_3)' \\ (fy_1)'' & (fy_2)'' & (fy_3)'' \end{vmatrix} = f^3 W^2 \neq 0.$$

Hence, functions fy_1, fy_2, fy_3 are the fundamental set of solutions of the equation

$$\frac{1}{f^3 W^2} \begin{vmatrix} fy_1 & fy_2 & fy_3 & u \\ (fy_1)' & (fy_2)' & (fy_3)' & u' \\ (fy_1)'' & (fy_2)'' & (fy_3)'' & u'' \\ (fy_1)''' & (fy_2)''' & (fy_3)''' & u''' \end{vmatrix} = 0. \tag{13}$$

Using that

$$\frac{1}{W^2} \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1''' & y_2''' & y_3''' \end{vmatrix} = -2p, \quad \frac{1}{W^2} \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1'' & y_2'' & y_3'' \\ y_1''' & y_2''' & y_3''' \end{vmatrix} = p' + q + p^2,$$

$$\frac{1}{W^2} \begin{vmatrix} y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \\ y_1''' & y_2''' & y_3''' \end{vmatrix} = -(q' + pq - r),$$

the equation (13) can be rewritten in the form

$$u''' + \left(2p - \frac{3f'}{f}\right) u'' + \left(\frac{6(f')^2}{f^2} - \frac{4f'p}{f} - \frac{3f''}{f} + (p' + q + p^2)\right) u' +$$
$$+ \left(-\frac{6(f')^3}{f^3} + \frac{4(f')^2 p}{f^2} + \frac{6f'f''}{f^2} - \frac{f'(p' + q + p^2)}{f} - \frac{2f''p}{f} - \frac{f'''}{f} + (q' + pq - r)\right) u = 0. \tag{14}$$

It is easy to show, that if $f \equiv 1$ then the equation (14) is equivalent to the equation (9), and if $f = \dfrac{1}{W}$ then the equation (14) is equivalent to the equation (2). Hence, Kim's equation and adjoint equation are the special cases of the equation (14).

Suppose that $f > 0 \ \forall t \in I$. Then $(\ln f)' = \dfrac{f'}{f}$, and the equation (14) can be written in the form

$$u''' + \Big(2p - 3(\ln f)'\Big)u'' + \Big(- 3(\ln f)'' + 3((\ln f)')^2 - 4p(\ln f)' + p' + q + p^2\Big)u' +$$
$$+ \Big(- (\ln f)''' - 2p(\ln f)'' + 3(\ln f)'(\ln f)'' - ((\ln f)')^3 + 2p((\ln f)')^2 -$$
$$- (\ln f)'(p' + q + p^2) + (q' + pq - r)\Big)u = 0.$$

And setting $(\ln f)' = g$, we get the equation

$$u''' + \Big(2p - 3g\Big)u'' + \Big(- 3g' + 3g^2 - 4pg + p' + q + p^2\Big)u' +$$
$$+ \Big(- g'' - 2pg' + 3gg' - g^3 + 2pg^2 - g(p' + q + p^2) + q' + pq - r\Big)u = 0. \quad (15)$$

Remark 5.1. If $x(t)$ is a nontrivial solution of the equation (1) which vanishes at $t = a$ ($x(a) = 0$), has a double zero at $t = \tau$ ($x(\tau) = x'(\tau) = 0$), then the solution $y(t)$ of equation (15), which has a double zero at $t = a$ ($y(a) = y'(a) = 0$) vanishes at $t = \tau$ ($y(\tau) = 0$).

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[3] W. J. Kim, Oscillatory Properties of Linear Third-Order Differential Equations, Proceedings of the American Mathematical Society, Vol. 26, No. 2. (Oct., 1970), pp. 286-293.

С. Смирнов. О связи между двойными и простыми нулями решений линейных дифференциальных уравнений третьего порядка.

Аннотация. Обсуждаются свойства нулей решений линейных дифференциальных уравнений третьего порядка. Рассматриваются сопряжённое уравнение и уравнение, которое имеет схожие свойства относительно распределения нулей. Представлено более общее уравнение, которое имеет те же свойства. Даются наглядные примеры и графики.

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S. Smirnovs. Par trešās kārtas lineāro diferenciālvienādojumu atrisinājumu divkāršam un parastam nullēm.

Anotācija. Tiek apspriestas trešās kārtas lineāru diferenciālvienādojumu atrisinājumu nuļļu īpašības. Tiek apskatīts saistītais vienādojums un vienādojums kurām ir līdzīgas īpašības attiecīgi pret nuļļu distribūciju. Tiek apskatīts vispārīgāks vienādojums kurām ir līdzīgas īpašības.

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