On nonlinear eigenvalue problems

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Summary. We consider the second order nonlinear eigenvalue problems depending on one or two parameters. First we are looking for positive solutions of equations $x'' = -f(x)$ and $x'' = -\lambda f(x)$, which are considered together with the Dirichlet boundary conditions $x(0) = 0,\ x(1) = 0,\quad (i)$. Function $f(x)$ is supposed to be convex. The relation between the parameter λ and the Nehari number $\lambda_0(0,1)$ is established ([1], [4]). Fučik like nonlinear problem is treated for equation $x'' = -\lambda f(x) + \mu g(x)$. We construct the set of points (λ, μ) such that this equation has a nontrivial normalized (by a condition $x'(0) = 1$) solution which satisfies the boundary conditions (i).

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1 Introduction

We consider autonomous equations of the type

$$x'' = -f(x), \tag{1}$$

$$x'' = -\lambda f(x), \tag{2}$$

and

$$x'' = -\lambda f(x) + \mu g(x), \tag{3}$$

where λ and μ are parameters, together with the boundary conditions

$$x(0) = 0, \quad x(1) = 0. \tag{4}$$

The functions $f(x)$ and $g(x)$ are positive valued continuous functions, defined on the half-axes $[0, +\infty)$ and $(-\infty, 0]$ respectively.

In the first case then $x''(t) \leq 0$ for a possible solution x and $x(t)$ is either zero or $x(t) > 0$ for $t \in (0,1)$. We are interested in the number of solutions for the problem (1), (4).

The problem (2), (4) was investigated by Laetsch [6] who was looking for λ such that the problem had a positive solution.

The third problem is in some sense generalization of the problem investigated by Laetsch and that of the classical two-parameter Fučik type eigenvalue problem.

2 The first problem

Consider the problem (1), (4), where $f : [0, +\infty) \to [0, +\infty)$ is a continuous function. It easily can be seen that if $f(0) = 0$, then $x(t) \equiv 0$ is a solution of the problem. Otherwise it has not the trivial solution.

Proposition 2.1 *Any nontrivial solution of the problem (1), (4) satisfies the condition*

$$x(t) > 0, \quad \forall t \in (0, 1), \tag{5}$$

it is symmetric with respect to the critical point $t = \frac{1}{2}$, *where it attains its maximal value.*

The proof can be found in [5].

Proposition 2.2 (Lemma 2.1 in [5]) *If* $f \in C^1([0, 1], [0, +\infty))$, *there are two positive solutions* u *and* v *of the problem (1), (4), then either* $u(t) < v(t)$ *for* $t \in (0, 1)$ *or vice versa.*

We would like to give slightly modified proof of this result.

Proof. Suppose that $v(t) > u(t)$ in some right vicinity of $t = 0$. Then $v'(0) > u'(0) > 0$. If $v(t) > u(t)$ for $t \in (0, \frac{1}{2}]$, then the proof is completed. Suppose this is not the case and there exists $\xi \in (0, \frac{1}{2}]$ such that $v(\xi) = u(\xi)$ and $v(t) > u(t)$ for $t \in (0, \xi)$. Notice that $0 \leq v'(\xi) < u'(\xi)$ then.

Introduce the primitive $F(z) = \int_0^z f(s)\, ds$. We multiply the equation $u'' + f(u) = 0$ by $2u'$ and integrate it over the interval $(0, \xi)$:

$$\int_0^\xi d(u'^2 + 2F(u)) = u'^2(\xi) + 2F(u(\xi)) - u'^2(0) - 2F(u(0)) = u'^2(\xi) + 2F(u(\xi)) - u'^2(0).$$

Repeating this procedure with respect to the function $v(t)$, one gets

$$\int_0^\xi d(v'^2 + 2F(v)) = v'^2(\xi) + 2F(v(\xi)) - v'^2(0).$$

Since $F(u(\xi)) = F(v(\xi))$ and $v'^2(\xi) < u'^2(\xi)$, it follows that $v'^2(0) < u'^2(0)$. A contradiction.

Proposition 2.3 *Suppose that the function* $f(x)$ *in (1) is convex, that is,*

$$\frac{f(v) - f(u)}{v - u} \leq \frac{f(w) - f(u)}{w - u} \tag{6}$$

for any $0 \leq u < v < w$.

Then, for any three solutions $u(t)$, $v(t)$ *and* $w(t)$ *of the equation (1) such that*

$$u(t) \leq v(t) \leq w(t) \quad \forall t \in [0, 1] \tag{7}$$

the function

$$\Phi(t) := (w - v)(v' - u') - (v - u)(w' - v') \tag{8}$$

is strictly increasing in $[0, 1]$ *function except when equation (1) is linear.*

The above proposition is Lemma 1 in [8], adapted for our purposes. The proof is short, so let us show it here.

Proof. It is easily confirmed that

$$\begin{aligned}\Phi'(t) = &\ (w-v)(v''-u'')-(v-u)(w''-v'')\\ = &\ (v-u)(f(w)-f(v))-(w-v)(f(v)-f(u)).\end{aligned}$$

Then, for any $0 \leq t_1 < t_2 \leq 1$,

$$\Phi(t_2)-\Phi(t_1) = \int_{t_1}^{t_2} [(v-u)(f(w)-f(v))-(w-v)(f(v)-f(u))]\, dt. \qquad (9)$$

Since $u < v < w$ and $f(x)$ is convex, we have

$$\frac{f(v)-f(u)}{v-u} \leq \frac{f(w)-f(u)}{w-u}$$

with equality only if the points $(u, f(u))$, $(v, f(v))$, $(w, f(w))$ lie on a straight line. It follows that the integrand in (9) is positive unless $f(x)$ is linear. This completes the proof.

Corollary 2.1 (Lemma II in [8]) *Let $f(x)$ be convex. If $u(t)$, $v(t)$, $w(t)$ are solutions of (1) such that $u(t) < w(t)$ and $v(t) < w(t)$ in $[0,1]$, then the curves $\xi = u(t)$ and $\eta = v(t)$ cannot intersect in $[0,1]$ more than once.*

Proof. If there are more points of intersection there will exist two points $0 \leq t_1 < t_2 \leq 1$ such that $u(t_1) = v(t_1)$ and $u(t_2) = v(t_2)$ and, say, $v(t) > u(t)$ in (t_1, t_2). If $\Phi(t)$ is the expression defined in (8), we then have

$$\Phi(t_1) = [w(t_1)-v(t_1)][v'(t_1)-u'(t_1)] > 0,$$

and

$$\Phi(t_1) = [w(t_2)-v(t_2)][v'(t_2)-u'(t_2)] < 0.$$

This contradicts Proposition 2.3 and thus proves the corollary.

Thus, the boundary value problem (1), (4) may have multiple positive solutions, if $f(x)$ is not convex. The trivial example can be given by $f(x) = \pi^2 x$ (infinitely many solutions). There are also equations of the form (1), which have countably many solutions which satisfy the boundary conditions (4).

Theorem 2.1 *If $f \in C^1$ in (1) is convex and $f(0) = 0$, then the problem (1), (4) has at most one positive solution.*

Proof. Follows from propositions 2.2 and 2.3.

3 The second problem

In this section we consider the eigenvalue problem (2), (4). Let us call λ by *the Laetsch parameter*. It can be seen easily that if this problem is solvable then λ is positive and the respective positive solution is symmetric with respect to the middle point $t = \frac{1}{2}$, where the maximal value is attained.

If $f(0) = 0$, then by the results of Section 1, only one positive solution is possible for any $\lambda > 0$, if $f \in C^1$ is convex.

Proposition 3.1 *Consider equation*

$$x'' = -\lambda x x^{2\alpha}, \quad \alpha > 0, \quad x \geq 0 \tag{10}$$

together with the boundary conditions (4). Let $\|x\| := \max_{[0,1]} x(t)$*, where* $x(t)$ *is the only positive solution of (3.1), (10) for a given* $\lambda > 0$.

Then

$$\|x\|^\alpha \cdot \sqrt{\lambda} = 2\sqrt{\alpha+1} \cdot A_\alpha, \tag{11}$$

where

$$A_\alpha = \int_0^1 \frac{d\xi}{\sqrt{1-\xi^{2\alpha+2}}}. \tag{12}$$

Proof. One gets, using the standard technique and following the notation by Laetsch [6], that

$$x'^2 = -\frac{1}{\alpha+1}\lambda x^{2\alpha+2} + \frac{1}{\alpha+1}\lambda \|x\|^{2\alpha+2}.$$

Then

$$\frac{dx}{dt} = \sqrt{\frac{1}{\alpha+1}\lambda\|x\|^{2\alpha+2} - \frac{1}{\alpha+1}\lambda x^{2\alpha+2}}$$

and, after integration in the interval of positivity of $x'(t)$, one has

$$\sqrt{\frac{\lambda}{\alpha+1}}t = \int_0^{x(t)} \frac{ds}{\sqrt{\|x\|^{2\alpha+2} - s^{2\alpha+2}}}$$

or

$$\frac{1}{2}\sqrt{\frac{\lambda}{\alpha+1}} = \frac{1}{\|x\|^\alpha}\int_0^1 \frac{d\xi}{\sqrt{1-\xi^{2\alpha+2}}} \tag{13}$$

Thus λ and $x(t)$ are connected by the relation

$$\|x\|^\alpha \cdot \sqrt{\lambda} = 2\sqrt{\alpha+1} \cdot A_\alpha, \tag{14}$$

which completes the proof.

4 Nehari equations

The Nehari theory, when restricted to autonomous equations, deals with equation

$$x'' = -xF(x^2), \tag{15}$$

where

- $F(s)$ is continuous for $s \geq 0$;
- $F(s) > 0$ for $s > 0$;
- there exists $\varepsilon > 0$ such that $\dfrac{F(s)}{s^\varepsilon}$ is non-decreasing.

Since F satisfies the condition (A3), the right side in (15) is a convex function, as well as the right side in

$$x'' = -\lambda x F(x^2), \tag{16}$$

where λ is supposed to be positive.

Let us consider equation (17), to which the Nehari theory is applicable. If $F(s) = s^\alpha$, then equation (17) takes the form of the Emden - Fowler equation

$$x'' = -\lambda x x^{2\alpha}, \quad \alpha > 0 \tag{17}$$

with a Laetsch parameter. Since the function $xx^{2\alpha}$ is continuously differentiable and convex for $x \geq 0$, there exists exactly one positive solution for any $\lambda > 0$. The relation between λ and $\|x\|$ was established in the preceding section.

On the other hand, for any $\lambda > 0$ there exists the so called Nehari number λ_1, which coincides with the minimal value of the functional

$$H(x) = \int_a^b \left[x'^2 - \lambda(1+\alpha)^{-1} x^{2+2\alpha}\right] dt \to \inf \tag{18}$$

over all positive solutions of the problem (17), (4). Generally the Nehari number λ_n is the minimal value of the functional over all solutions of the problem (17), (4), which have exactly $n-1$ zeros in (a, b). Since there is only one positive solution, it furnishes the minimal value to the functional. This solution and the value of the functional can be computed in order to get λ_1.

In the next proposition the relation between λ and λ_1 is established.

Proposition 4.1 *The Nehari number λ_1 and the Laetsch parameter λ are connected by the relation*

$$\lambda_1 = \frac{\alpha(\alpha+1)^{\frac{1}{\alpha}}}{\alpha+2} (2A_\alpha)^{\frac{2\alpha+2}{\alpha}} \lambda^{-\frac{1}{\alpha}} \tag{19}$$

where A_α is given in (12).

Proof. It was shown in [4] that the Nehari numbers λ_n for the problem

$$x'' = -\frac{k}{(pt+q)^{2\varepsilon+4}} x^{2\varepsilon} x, \quad x(a) = 0, \quad x(b) = 0$$

where $pt + q > 0$ in the interval $[a; b]$ $(0 \leq a < b)$ and $k > 0$, are given by

$$\lambda_n(a,b) = \frac{\varepsilon(\varepsilon+1)^{\frac{1}{\varepsilon}}}{\varepsilon+2} (2A_\varepsilon n)^{\frac{2\varepsilon+2}{\varepsilon}} k^{-\frac{1}{\varepsilon}} \left(\frac{(pa+q)(pb+q)}{b-a} \right)^{\frac{\varepsilon+2}{\varepsilon}}, \tag{2}$$

where $A_\varepsilon = \int\limits_0^1 \frac{du}{\sqrt{1-u^{2\varepsilon+2}}}$. For the specific choice of $k = \lambda$, $pt + q \equiv 1$, $a = 0$, $b = 1$, $n = 1$ one obtains the formula (19).

5 The third problem in a specific form

Consider now the problem (3), (4), which can be written also as

$$x'' = \begin{cases} -\lambda f(x), & \text{if} \quad x \geq 0 \\ \mu g(x), & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0. \tag{20}$$

If $f(x) = \max\{x, 0\}$ and $g(x) = \max\{-x, 0\}$, then one has the famous Fučik problem. The nonlinear generalization of this problem was considered in [7] for cubic nonlinearities. The spectrum obtained was very similar to the classical Fučik spectrum, at least with respect to the structure.

Let us consider the specific case of

$$x'' = \begin{cases} -\lambda x^{2\alpha} x, & \text{if} \quad x \geq 0 \\ -\mu x^{2\beta} x, & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0, \tag{21}$$

where $\lambda \geq 0$ and $\mu \geq 0$; $\alpha > 0$ un $\beta > 0$.

In order to get reasonable problem we have to impose the normalization condition $|x'(0)| = 1$.

Definition. By the Fučik spectrum for the problem (20) is meant a set of all points $(\lambda \geq 0, \mu \geq 0)$ such that the problem has a nontrivial solution.

Consider the auxiliary problem

$$x'' = -\lambda x^{2\alpha} x, \quad x(0) = 0, \ x'(0) = 1. \tag{22}$$

Let T_α be the first zero of $x(t)$ for $t > 0$. Introduce the notation $\|x\| = \max\{x(t) : \ 0 \leq t \leq T_\alpha\}$ and notice that $\|x\| = x(T_\alpha/2)$ and $x'(T_\alpha) = -1$.

Lemma 5.1 *It is true that*

$$T_\alpha = 2\frac{(\alpha+1)^{\frac{1}{2\alpha+2}}}{\lambda^{\frac{1}{2\alpha+2}}} A_\alpha \tag{23}$$

where A_α is given by (12).

Proof. By standard computation, multiplying both sides of the equation in (22) by $2x'$ and integrating from 0 to $T_\alpha/2$.

Therefore $T_\alpha \to 0$ as $\lambda \to +\infty$.

Theorem 5.1 *The spectrum of the problem (21) consists of a set of curves F_0^+, F_0^-, F_{2i}^+, F_{2i}^-, and F_{2i-1}^+, F_{2i-1}^-, $(i = 1, 2, \ldots)$, given by:*

$$F_0^+ = \Big\{\big((2A_\alpha)^{2\alpha+2}(\alpha+1);\mu\big):\quad \mu \geq 0\Big\}, \tag{24}$$

$$F_0^- = \Big\{\big(\lambda;(2A_\beta)^{2\beta+2}(\beta+1)\big):\quad \lambda \geq 0\Big\}, \tag{25}$$

$$F_{2i-1}^+ = \left\{(\lambda;\mu):\quad \frac{2iA_\alpha(\alpha+1)^{\frac{1}{2\alpha+2}}}{\lambda^{\frac{1}{2\alpha+2}}} + \frac{2iA_\beta(\beta+1)^{\frac{1}{2\beta+2}}}{\mu^{\frac{1}{2\beta+2}}} = 1\right\}, \tag{26}$$

$$F_{2i-1}^- = \left\{(\lambda;\mu):\quad \frac{2iA_\beta(\beta+1)^{\frac{1}{2\beta+2}}}{\mu^{\frac{1}{2\beta+2}}} + \frac{2iA_\alpha(\alpha+1)^{\frac{1}{2\alpha+2}}}{\lambda^{\frac{1}{2\alpha+2}}} = 1\right\}, \tag{27}$$

$$F_{2i}^+ = \left\{(\lambda;\mu):\quad \frac{2(i+1)A_\alpha(\alpha+1)^{\frac{1}{2\alpha+2}}}{\lambda^{\frac{1}{2\alpha+2}}} + \frac{2iA_\beta(\beta+1)^{\frac{1}{2\beta+2}}}{\mu^{\frac{1}{2\beta+2}}} = 1\right\}, \tag{28}$$

$$F_{2i}^- = \left\{(\lambda;\mu):\quad \frac{2(i+1)A_\beta(\beta+1)^{\frac{1}{2\beta+2}}}{\mu^{\frac{1}{2\beta+2}}} + \frac{2iA_\alpha(\alpha+1)^{\frac{1}{2\alpha+2}}}{\lambda^{\frac{1}{2\alpha+2}}} = 1\right\}, \tag{29}$$

where

$$A_\alpha = \int_0^1 \frac{dt}{\sqrt{1-t^{2\alpha+2}}}, \quad A_\beta = \int_0^1 \frac{dt}{\sqrt{1-t^{2\beta+2}}}.$$

Corollary 5.1 *The branches F_{2i-1}^+ and F_{2i-1}^- coincide $(i = 1, 2, \ldots)$.*

Proof of the theorem. Consider the equation in (21) together with the initial conditions

$$x(0) = 0, \quad x'(0) = 1. \tag{30}$$

We are looking for a positive solution of the problem (21). This solution solves the auxiliary problem (22), where λ can be found from the relation $T_\alpha = 1$. It follows from (23) that

$$\lambda = (2A_\alpha)^{2\alpha+2}(\alpha+1).$$

Thus the expression for (24) is valid.

In order to get the formula for the curve F_0^- one should look for a negative solution of the problem

$$x'' = -\mu x^{2\beta}x, \quad x(0) = 0, \quad x(1) = 0.$$

This is equivalent to the problem

$$y'' = -\mu y^{2\beta}y, \quad y(0) = 0,\ y(1) = 0,\ y(t) \text{ is positive in } (0,1).$$

Up to the notation this is the above problem and one gets by resolving the equation $T_\mu = 1$ that

$$\mu = (2A_\beta)^{2\beta+2}(\beta + 1).$$

Thus the expression (25) is valid.

In order to prove (26) one should consider the equation in (21) together with the initial conditions (30). We are looking for solutions which are zero at the end points and have exactly $2i - 1$ zeros t_k in $(0, 1)$. Notice that all these zeros are simple and $|x'(t_k)| = 1$. In the intervals of positivity a solution $x(t)$ satisfies $x'' = -\lambda x^{2\alpha} x$ and in the intervals of negativity $x(t)$ is a solution of $x'' = -\mu x^{2\beta} x$. The key relation is

$$T_\alpha + T_\beta + \ldots + T_\alpha + T_\beta = iT_\alpha + iT_\beta = 1. \tag{31}$$

Formula (26) follows from (31) and (23).

Similarly (27), (28) and (29) follow from the key relations

$$T_\beta + T_\alpha + \ldots + T_\beta + T_\alpha = iT_\beta + iT_\alpha = 1,$$

$$T_\alpha + T_\beta + \ldots + T_\beta + T_\alpha = (i + 1)T_\alpha + iT_\beta = 1$$

and

$$T_\beta + T_\alpha + \ldots + T_\alpha + T_\beta = iT_\alpha + (i + 1)T_\beta = 1.$$

6 Unified approach to superlinear and sublinear cases

Consider the Dirichlet problem $[(i, \lambda, \alpha), (j, \mu, \beta)]$:

$$x'' = \begin{cases} -\lambda f_\alpha^i(x), & \text{if} \quad x \geq 0, \\ -\mu f_\beta^j(x), & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0,$$

where $i, j \in \{\uparrow; \downarrow\}$, $\alpha > 0$, $\beta > 0$ and

$$f_\alpha^\uparrow(x) = x^{2\alpha} x, \quad f_\beta^\downarrow(x) = |x|^{\frac{1}{2\alpha+1}} \operatorname{sign} x.$$

The notation above allows to describe in a unified manner the following cases:

- $\uparrow + \uparrow$ super+super

$$x'' = \begin{cases} -\lambda f_\alpha^\uparrow(x), & \text{if} \quad x \geq 0, \\ -\mu f_\beta^\uparrow(x) & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0,$$

- $\uparrow + \downarrow$ super+sub

$$x'' = \begin{cases} -\lambda f_\alpha^\uparrow(x), & \text{if} \quad x \geq 0, \\ -\mu f_\beta^\downarrow(x) & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0,$$

- $\downarrow + \uparrow$ sub+super

$$x'' = \begin{cases} -\lambda f_\alpha^\downarrow(x), & \text{if} \quad x \geq 0, \\ -\mu f_\beta^\uparrow(x) & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0,$$

- $\downarrow + \downarrow$ sub+sub

$$x'' = \begin{cases} -\lambda f_\alpha^\downarrow(x), & \text{if} \quad x \geq 0, \\ -\mu f_\beta^\downarrow(x) & \text{if} \quad x \leq 0, \end{cases} \qquad x(0) = x(1) = 0.$$

Let us denote:

$$A_\alpha^\uparrow = \int_0^1 \frac{dt}{\sqrt{1 - t^{2\alpha+2}}}, \quad A_\alpha^\downarrow = \int_0^1 \frac{dt}{\sqrt{1 - t^{\frac{2\alpha+2}{2\alpha+1}}}}.$$

$$T_{\lambda,\alpha}^\uparrow = \frac{2A_\alpha^\uparrow(\alpha+1)^{\frac{1}{2\alpha+2}}}{\lambda^{\frac{1}{2\alpha+2}}}, \quad T_{\lambda,\alpha}^\downarrow = \frac{2A_\alpha^\downarrow \left(\frac{\alpha+1}{2\alpha+1}\right)^{\frac{2\alpha+1}{2\alpha+2}}}{\lambda^{\frac{2\alpha+1}{2\alpha+2}}}.$$

Theorem 6.1 *The Fuchik spectrum for the problem* $[(i, \lambda, \alpha), (j, \mu, \beta)]$ *consists of the curves:*

No zeros in the interval $(0; 1)$.

$${}^{\uparrow,j}F_0^+ = \left\{ \left((T_{1,\alpha}^\uparrow)^{2\alpha+2}; \mu\right) : \quad \mu \geq 0 \right\}.$$

$${}^{i,\uparrow}F_0^- = \left\{ \left(\lambda; (T_{1,\beta}^\uparrow)^{2\beta+2}\right) : \quad \lambda \geq 0 \right\}.$$

$${}^{\downarrow,j}F_0^+ = \left\{ \left((T_{1,\alpha}^\downarrow)^{\frac{2\alpha+2}{2\alpha+1}}; \mu\right) : \quad \mu \geq 0 \right\}.$$

$${}^{i,\downarrow}F_0^- = \left\{ \left(\lambda; (T_{1,\beta}^\downarrow)^{\frac{2\beta+2}{2\beta+1}}\right) : \quad \lambda \geq 0 \right\}.$$

Odd number of zeros $2k-1$ *in the interval* $(0; 1)$.

$${}^{i,j}F_{2k-1}^+ = \left\{ (\lambda; \mu) : \quad kT_{\lambda,\alpha}^i + kT_{\mu,\beta}^j = 1 \right\}.$$

$${}^{i,j}F_{2k-1}^- = \left\{ (\lambda; \mu) : \quad kT_{\lambda,\alpha}^i + kT_{\mu,\beta}^j = 1 \right\}.$$

Here $k = 1, 2, \ldots$. *It is clear that* ${}^{i,j}F_{2i-1}^+ = {}^{i,j}F_{2i-1}^-$.

Even number of zeros $2k$ *in the interval* $(0; 1)$.

$${}^{i,j}F_{2i}^+ = \left\{ (\lambda; \mu) : \quad (k+1)T_{\lambda,\alpha}^i + kT_{\mu,\beta}^j = 1 \right\}.$$

$${}^{i,j}F_{2i}^- = \left\{ (\lambda; \mu) : \quad kT_{\lambda,\alpha}^i + (k+1)T_{\mu,\beta}^j = 1 \right\}.$$

Here $k = 1, 2, \ldots$.

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А. Грицанс, Ф. Садырбаев. О нелинейных задачах на собственные значения.

Аннотация. Рассматриваются нелинейные уравнения с одним и двумя параметрами. Сначала приводятся условия существования положительного решения уравнений $x'' = -f(x)$ и $x'' = -\lambda f(x)$, рассматриваемых вместе с краевыми условиями $x(0) = 0,\ x(1) = 0, \quad (i)$. Функция $f(x)$ предполагается вогнутой. Устанавливается соотношение между параметром λ и числом Нехари $\lambda_1(0,1)$ ([1], [4]). Для уравнения $x'' = -\lambda f(x) + \mu g(x)$ рассматривается нелинейная задача типа Фучика. Описывается множество точек (λ, μ) таких, что существует нетривиальное нормированное ($x'(0) = 1$) решение, удовлетворяющее условиям (i).

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A. Gricāns, F. Sadirbajevs. Par nelineāram īpašvērtību problēmām.

Anotācija. Tiek apskatīti nelineāri diferenciālvienādojumi ar vienu un diviem parametriem. Sākumā doti nosacījumi pozitīva atrisinājuma eksistencei vienādojumiem $x'' = -f(x)$ un $x'' = -\lambda f(x)$, kuri tiek apskatīti kopā ar robežnosacījumiem $x(0) = 0$, $x(1) = 0$. Iegūta sakarība starp parametru λ un Nehari skaitli $\lambda_1(0,1)$ ([1], [4]). Vienādojumam $x'' = -\lambda f(x) + \mu g(x)$ tiek pētīta nelineāra Fučika tipa problēma. Tiek aprakstīta tāda punktu (λ, μ) kopa, ka eksistē netriviāls normēts ($x'(0) = 1$) atrisinājums.

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