26th International Conference on

MATHEMATICAL MODELLING AND ANALYSIS

May 30 – June 2, 2023, Jurmala, Latvia

ABSTRACTS

UNIVERSITY OF LATVIA

Latvian Mathematical Society
Faculty of Physics, Mathematics and Optometry, University of Latvia
Institute of Mathematics and Computer Science, University of Latvia
Vilnius Gediminas Technical University

26th International Conference

Mathematical Modelling and Analysis

May 30 – June 2, 2023, Jurmala, Latvia

Abstracts

University of Latvia
Riga 2023

Conference Organizers:

Latvian Mathematical Society
Faculty of Physics, Mathematics and Optometry, University of Latvia
Institute of Mathematics and Computer Science, University of Latvia
Vilnius Gediminas Technical University

International Scientific Committee:

R. Ciegis (Chairman, Lithuania)
S. Asmuss (Latvia)
I. Bula (Latvia)
F. Gaspar (Spain)
A. Gritsans (Latvia)
U. Hämarik (Estonia)
O. Iliev (Germany)
J. Janno (Estonia)
A. Kolishkins (Latvia)
A. Pedas (Estonia)
K. Pileckas (Lithuania)
M. Radziunas (Germany)
M. Ragulskis (Lithuania)
A. Reinfelds (Latvia)
F. Sadyrbaev (Latvia)
M. Sapagovas (Lithuania)
A. Sostaks (Latvia)
V. Starikovicius (Lithuania)
A. Stikonas (Lithuania)
U. Strautins (Latvia)

Organizing Committee:

U. Strautins (Chairman)
R. Ciegis (Vice-chairman)
S. Asmuss
R. Bets
N. Budkina
I. Bula
O. Grigorenko
A. Kolishkins
M. Marinaki
A. Reinfelds
S. Smirnovs
I. Uljane

https://doi.org/10.22364/JZYM4660

ISBN 978-9934-36-011-4
ISBN 978-9934-36-012-1 (PDF)

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

BARTLETT CORRECTIONS FOR QUANTILE INFERENCE WITH EMPIRICAL LIKELIHOOD

REINIS ALKSNIS, JANIS VALEINIS

University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: `reinis.alksnis@lu.lv`

Empirical likelihood is a nonparametric approach to statistical inference which preserves many desirable properties of its parametric counterpart maximum likelihood. As in the parametric case the test procedures usually rely on the fact that statistic has an asymptotic chi square distribution. It is well known that in the case of maximum likelihood a scalar multiplier, known as Bartlett correction, can be derived which improves the accuracy of this approximation. Somewhat surprisingly it turns out that in many circumstances empirical likelihood can be Bartlett corrected as well. This is not only interesting from the theoretical point of view, since it implies that there is a deeper relation between parametric and empirical likelihoods, but it is also practically useful because we can derive statistical tests that are more accurate, especially when the sample sizes are small.

In a seminal paper DiCiccio et al. [1] showed that empirical likelihood can be Bartlett corrected when the parameter of interest can be expressed as a smooth function of means. Afterwards Bartlett corrections have been derived in many empirical likelihood related papers for many other models. In one such paper Chen and Hall [2] derived Bartlett corrections for quantile inference when kernel smoothing is applied to empirical likelihood. Undoubtedly this is a notable theoretical result, however from the simulation analysis the practical significance of the corrections seems questionable. It appears that when kernel smoothing is applied to empirical likelihood Bartlett corrections make almost no improvement. Hence the objective of this study is to investigate whether Bartlett corrections make a substantial improvement when confidence intervals for quantiles are constructed with smoothed empirical likelihood. In order to accomplish this the derivations made by Chen and Hall are reconstructed and an extensive simulation analysis is conducted.

REFERENCES

[1] T. DiCiccio, P. Hall, J. Romano. Empirical likelihood is Bartlett-correctable. *The Annals of Statistics*, **19** (2):1053–1061, 1991.

[2] S.X. Chen, P. Hall. Smoothed empirical likelihood confidence intervals for quantiles. *The Annals of Statistics*, **21** (3):1166–1181, 1993.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

NUMERICAL ASPECTS OF MODULATION INSTABILITY

SHALVA AMIRANASHVILI

Weierstrass Institute

Mohrenstrasse 39, 10117 Berlin, Germany

E-mail: `shalva.amiranashvili@wias-berlin.de`

Evolution of a nonlinear wave may result in modulation instability (MI): a spontaneous growth of the small amplitude modulations that transforms the initial wave in a sequence of pulses. MI was detected in numerous nonlinear wave systems, in optical fibers it is described by the generalized nonlinear Schrödinger equation (GNLSE) for the complex wave envelope $\psi(z,\tau)$

$$i\partial_z\psi + \hat{D}\psi + \gamma(|\psi|^2 - P_0)\psi = 0, \quad \text{where} \quad \hat{D} : e^{-i\nu\tau} \mapsto D(\nu)e^{-i\nu\tau}. \tag{1}$$

GNLSE "accepts" the initial temporal pulse shape $\psi(0,\tau)$ and "returns" the resulting pulse shape along the fiber, $\psi(z,\tau)$, where z is the propagation distance. The so-called *dispersion operator* $\hat{D}$ is usually represented by a polynomial $D(\nu)$ and is then a differential operator, which can be written as $\hat{D} = D(i\partial_\tau)$. In the long run, MI contributes to the formation of solitons, generation of wave turbulence, optical supercontimuum, formation of extreme waves, and so on, see [1].

This contribution deals with the numerical aspects of MI. Namely, the small wave modulations are so fundamentally simple, that it is possible to study them directly for the numerical schemes that solve GNLSE, such as the split-step Fourier method. A requirement that the numerical solution reproduces MI, while avoiding the nonphysical numerical instabilities, provides information on the applicability of the method. We derive the following statement

Theorem. *If GNLSE* (1) *is solved numerically using the split-step Fourier method with the temporal resolution $\Delta\tau$, the evolution step h shall obey the inequality*

$$h < h_{max} = \frac{\pi}{\|D\| + 2|\gamma|P_0}, \quad \|D\| = \max_{|\omega_{b,r}| < \frac{\pi}{\Delta\tau}} \left| \frac{D(\omega_b) + D(\omega_r)}{2} - D\left(\frac{\omega_b + \omega_r}{2}\right) \right|. \tag{2}$$

Violation of the inequality (2) leads to slow excitation of the spurious resonant wave interactions, which may completely spoil the numerical solution for a realistic fiber length. The inequality is especially important to meet when an envelope equation has to be applied in a wide spectral window, e.g., because of the spectral broadening.

REFERENCES

[1] G. P. Agrawal. *Nonlinear Fiber Optics.* New York, 2007.

[2] F. Severing, U. Bandelow, Sh. Amiranashvili. Spurious four-wave mixing processes in generalized nonlinear Schrödinger equations. *Journal of Lightwave Technology*, 2023. (submitted)

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

COMPUTATION OF NUMERICALLY EXACT STATIONARY AND MOVING POLAROBREATHERS

JĀNIS BAJĀRS[1], JUAN F. R. ARCHILLA[2]

[1] *Faculty of Physics, Mathematics and Optometry, University of Latvia*

Jelgavas street 3, Riga, LV-1004, Latvia

[2] *Group of Nonlinear Physics, Universidad de Sevilla*

ETSII, Avda Reina Mercedes s/n, 41012-Sevilla, Spain

E-mail: `janis.bajars@lu.lv`

In this work, we develop a numerical algorithm based on nonlinear least squares with constraints to obtain numerically exact stationary and moving polarobreather solutions [1], i.e., discrete breathers carrying charge, electron or hole, in semiclassical crystal lattice models. For the time integration of the nonseparable canonical Hamiltonian system, we consider a recently proposed structure-preserving numerical method that exactly preserves the total charge probability [2]. The essential properties of stationary and moving polarobreathers are deduced from the lattice and charge variables spectra in the frequency-momentum space, where the wave-characterization of exact discrete breathers has been put forward in [3; 4] in one and two-dimensional crystal lattice models, respectively. Authors in [1] have extended the breather theory to polarobreathers in semiclassical models. The spectrum of approximate polarobreather allows for determining the necessary parameters for the proposed numerical algorithm to compute exact polarobreathers, i.e., the fundamental time or period, the charge energy shift value, and the spatial step for moving solutions. The spectra of exact polarobreathers become extremely simple and easy to interpret. Understanding the polarobreather spectrum is fundamental to interpreting possible signatures of localized nonlinear waves in the physical spectra of real crystals. With our proposed methodology, we also solve the problem that the charge frequency is not an observable, whereas the frequency of the charge probability, which is related to the discrete breather frequency, is an observable. Mathematical details of the methods, numerical results, and spectra of polarobreathers will be presented at the conference.

Acknowledgment. J. Bajārs acknowledges financial support by the specific support objective activity 1.1.1.2. "Post-doctoral Research Aid" of the Republic of Latvia (Project No. 1.1.1.2/VIAA/4/20/617 "Data-Driven Nonlinear Wave Modelling"), funded by the European Regional Development Fund (project id. N. 1.1.1.2/16/I/001). J.F.R. Archilla acknowledges support from projects MICINN PID2019-109175GB-C22 and Junta de Andalucía US-1380977, and a travel grant from VIIPPITUS 2023.

REFERENCES

[1] J.F.R. Archilla, J. Bajārs. Spectral properties of exact polarobreathers in semiclassical systems. *Axioms*, **12** (5):437, 2023.

[2] J. Bajārs, J.F.R. Archilla. Splitting methods for semi-classical Hamiltonian dynamics of charge transfer in nonlinear lattices. *Mathematics*, **10** (19):3460, 2022.

[3] J.F.R. Archilla, Y. Doi, M. Kimura. Pterobreathers in a model for a layered crystal with realistic potentials: Exact moving breathers in a moving frame. *Physical Review E*, **100** (2):022206, 2019.

[4] J. Bajārs, J.F.R. Archilla. Frequency-momentum representation of moving breathers in a two dimensional hexagonal lattice. *Physica D: Nonlinear Phenomena*, **441** :133497, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON THE LAPLACE TRANSFORM OF THE RIEMANN ZETA-FUNCTION

AIDAS BALČIŪNAS

Vilnius University, Mathematical Faculty

Naugarduko 24, Vilnius LT-03225, Lithuania

E-mail: aidas.balciunas@mif.vu.lt

Let $s = \sigma + it$ be a complex variable. The Laplace transform $\mathfrak{L}(s, f)$ of a function f is defined by

$$\mathfrak{L}(s, f) = \int_0^\infty f(x)e^{-sx}dx$$

provided that the integral exists for $\sigma > \sigma_0$ with some $\sigma_0 \in \mathbb{R}$. Denote by $\zeta(s)$ the Riemann zeta-function.

This report is a generalized case of [1], and is devoted to the Laplace transform

$$\mathfrak{L}\left(s, |\zeta|^{2k}\right) = \int_0^\infty \left|\zeta\left(\frac{1}{2} + ix\right)\right|^{2k} e^{-sx}dx,$$

where $k \in \mathbb{N}$. In [1], the case $k = 2$ has been considered. In the report, we obtain explicit formulae involving some elementary functions for $\mathfrak{L}\left(s, |\zeta|^{2k}\right)$ with arbitrary $k \in \mathbb{N}$.

REFERENCES

[1] A. Ivič. The Laplace transform of the fourth moment of the zeta-function. *Univ. Beograd. Publ. Eelektrotehn. Scr. Mat.*, **11** :41–48, 2000.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON PARAMETERIZED METRIC AND ITS APPLICATIONS IN COMBINATORICS ON WORDS

RAIVIS BĒTS

Institute of Mathematics and Computer Science, University of Latvia

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `raivis.bets@lu.lv`

In the literature there are several kinds of metrics on the set X of infinite words. The first one is defined as follows, see e.g. [1]. Let $x, y \in X$ be infinite words. Then

$$\rho(x,y) = \begin{cases} 0 & \text{if } x = y, \\ 2^{-n} & \text{otherwise where } n = \min\{i : x_i \neq y_i\}, \end{cases}$$

is a metric on the set of infinite words, where $\rho : X \times X \to [0; 1]$.

Another known definition of a metric on the set of infinite words is introduced as follows, see e.g. [2]. Let $x, y \in X$ be infinite words, and let for a given $i \in \mathbb{N}$ the number χ_i be defined by:

$$\chi_i(x,y) = \begin{cases} 0 & \text{if } x_i = y_i \text{ where } i \text{ is the } i\text{-th coordinate of the word,} \\ 1 & \text{if } x_i \neq y_i \text{ where } i \text{ is the } i\text{-th coordinate of the word.} \end{cases}$$

Now let

$$\sigma(x,y) = \sum_{i=0}^{\infty} \frac{1}{2^i} \chi_i(x,y).$$

Then $\sigma : X \times X \to [0, 2]$ is a metric (actually ultrametric) on the set of all infinite words. We give some counterexamples for both these metrics, where results contradict our intuition.

We suggest to use parameterized metric, see [3], and define a parameterized metric, which is suitable for comparing distance between two infinite words (also comparing their prefixes) defining:

$$P(x,y,t) = \frac{2}{\lceil t \rceil \cdot \lceil t+1 \rceil} \sum_{i=1}^{\lceil t \rceil} \chi_i(x,y) \left(\lceil t+1 \rceil - i \right).$$

We give some results for counterexamples, general cases and particularly for periodic words.

Acknowledgment. This work is supported by the ERDF PostDoc Latvia project Nr. 1.1.1.2/16/I/001 under agreement Nr. 1.1.1.2/VIAA/4/20/706/ "Applications of Fuzzy Pseudometrics in Combinatorics on Words".

REFERENCES

[1] J.P. Allouche, J. Shallit. *Automatic Sequences: Theory, Applications, Generalizations.* Cambridge Univ. Press, 2003.

[2] R.A. Holmgren. *A First Course in Discrete Dynamical Systems.* Springer-Verlag, 2nd edition, 2000.

[3] R. Bēts, A. Šostak, E.M. Miķelsons. Parameterized metrics and their applications in word combinatorics. In: *Proc. of the 6th Intern. Conference IPMU, Milan, Italy, 2022*, Communications in Computer and Information Science, vol. 1601, Springer, Cham., 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

AMPLITUDE EVOLUTION EQUATIONS FOR THE ANALYSIS OF NON-ISOTHERMAL INCOMPRESSIBLE FLOWS

NATALJA BUDKINA, ANDREI KOLYSHKIN

Riga Technical University

Zunda embankment 10, Riga LV-1048, Latvia

E-mail: `natalja.budkina@rtu.lv, andrejs.koliskins@rtu.lv`

The paper deals with weakly nonlinear approach to stability analysis of non-isothermal flows under the Boussinesq approximation. We consider a tall vertical channel bounded by two infinitely long parallel planes. The channel is assumed to be closed at infinity. Steady convective flow in a vertical direction can be generated by two factors: (a) different constant wall temperatures and/or (b) internal heat sources induced in the fluid. We assume that there exists a steady base flow in the vertical direction depending on the transverse coordinate.

Imposing small perturbations on the flow and linearizing the governing equations in the neighborhood of the base flow we obtain linear stability equations which are coupled with boundary conditions. The corresponding boundary value problem is solved numerically and the critical values of the Grashof number (the parameter characterizing the flow) are found. Linear stability theory gives the conditions when the given steady flow becomes unstable but it cannot be used to analyze the development of instability above the threshold. In this case one can use weakly nonlinear theory. It is assumed that the Grasshof number is slightly above the critical value so that the base flow is unstable but the growth rate of a perturbation is small.

Method of multiple scales [1], [2] is used in the paper to develop amplitude evolution equation for the most unstable mode. It is shown that for the case of planar perturbations the evolution equation is the complex Ginzburg-Landau equation. The coefficients of the equation are expressed in terms of integrals containing solutions of several boundary value problems.

Acknowledgment. This research was funded by the Latvian Council of Science project No. lzp-2020/1-0076.

REFERENCES

[1] K. Stewartson, J.T. Stuart. A nonlinear instability theory for a wave system in plane Poiseuille flow. *Journal of Fluid Mechanics*, **48** :529–545, 1971.

[2] A.A. Kolyshkin, M.S. Ghidaoui. Stability analysis of shallow wake flows. *Journal of Fluid Mechanics*, **494** :355–377, 2003.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

THE INVESTIGATION OF DATA TEMPORAL DELAYS IMPACT ON CLASSIFICATION PERFORMANCE FOR CHURN PREDICTION IN TELECOMMUNICATIONS

ANDREJ BUGAJEV, RIMA KRIAUZIENĖ

Vilnius Gediminas Technical University

Saulėtekio al. 11, LT-10223 Vilnius

E-mail: `andrej.bugajev@vilniustech.lt`

The field of Machine Learning (ML) is focused on developing models based on standardized benchmarks, which are used to compare performance among developers. The benchmarks rely on datasets that are either based on real data or synthesized according to observations from real data. ML models often aim to predict outcomes through either classification or regression. However, there are important questions that are often overlooked, which can result in discrepancies between a model's performance during research and its performance in real-world applications. In this talk, we will address some of these key issues, including:

- The behavior patterns of observed objects can change over time, which can cause delays between prediction and training intervals and affect the accuracy of results. While this delay cannot be avoided, it must be considered when optimizing the practical outcome of the technology, as demonstrated in [1].
- The classic approach of splitting a dataset into training, validation, and testing parts does not allow for precise estimation of a model's performance in a real-world scenario. Therefore, an additional set of data must be included, which we will refer to as "true testing data". This data must be qualitatively different from the training/validation data and should consist of information from the future that can simulate and evaluate the model's application with a delay.
- The effect of delay in training and prediction can vary depending on the specific time window, so it's important to observe different time windows.

Throughout this discussion, we will provide an illustrative case study of churn prediction in telecommunications. In this context, a churner is defined as a user who has stopped using a specific service, such as telecommunication services from a specific operator. While the most common definition of a churner in telecommunications is a client who has not generated any revenue for three months, alternative definitions may lead to better prediction or retention results, as noted in [1].

REFERENCES

[1] A. Bugajev, R. Kriauzienė, O. Vasilecas, V. Chadyšas. The impact of churn labelling rules on churn prediction in telecommunications. *Informatica*, **33** (2):247-277, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ONE NEURON MODEL AS CHAOTIC ATTRACTOR

INESE BULA

Department of Mathematics, University of Latvia
Jelgavas street 3, Riga LV-1004, Latvia
Institute of Mathematics and Computer Science, University of Latvia
Raina blvd. 29, Riga LV-1459, Latvia
E-mail: inese.bula@lu.lv

We study the following non-autonomous piecewise linear difference equation

$$x_{n+1} = \beta_n x_n - g(x_n), \quad n = 0, 1, 2, ..., \tag{1}$$

where $(\beta_n)_{n=0}^{\infty}$ is a period two or three sequence

$$\beta_n = \begin{cases} \beta_0, & \text{if } n = 2k, \\ \beta_1, & \text{if } n = 2k+1, \end{cases} \quad \text{or} \quad \beta_n = \begin{cases} \beta_0, & \text{if } n = 3k, \\ \beta_1, & \text{if } n = 3k+1, \\ \beta_2, & \text{if } n = 3k+2, \end{cases} \quad k = 0, 1, 2, ...,$$

and g is the piecewise constant function with McCulloch-Pitts nonlinearity

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \tag{2}$$

In [1] a difference equation $x_{n+1} = \beta x_n - g(x_n), \quad n = 0, 1, 2, ...$, with (2) was analyzed as a single neuron model where $\beta > 0$ is an internal decay rate and g is a signal function. In [2; 3; 4] we have been studied this model where $(\beta_n)_{n=0}^{\infty}$ is a period two and three sequences. We investigated the periodic behavior and stability of the solutions depending on the size of $(\beta_n)_{n=0}^{\infty}$.

Now we show that for certain values of the coefficients there exists an attracting interval for which the model is chaotic. On the other hand, if the initial value is chosen outside the mentioned attracting interval, then the solution of the difference equation either increases to positive infinity or decreases to negative infinity. The properties of chaos (sensitivity to initial conditions) can be used in random number generation as well as in cryptography.

The results are obtained in collaboration with M.A. Radin, Rochester Institute of Technology, U.S.A.

REFERENCES

[1] Z. Zhou. Periodic orbits on discrete dynamical systems. *Computers and Mathematics with Applications*, **45** :1155–1161, 2003.

[2] I. Bula, M.A. Radin. Periodic orbits of a neuron model with periodic internal decay rate. *Appl. Math. Comput.*, **266** :293–303, 2015.

[3] I. Bula, M.A. Radin, N. Wilkins. Neuron model with a period three internal decay rate. *Electron. J. Qual. Theory Differ. Equ.*, **46** :1–19, 2017.

[4] I. Bula, M.A. Radin. Eventually periodic solutions of single neuron model. *Nonlinear Anal. Model. Control*, **25** :903–918, 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

EFFICIENT NUMERICAL ALGORITHMS FOR PARTIALLY DIMENSION REDUCED NONLOCAL HEAT CONDUCTION PROBLEMS

RAIMONDAS ČIEGIS

Vilnius Gediminas Technical University

Saulėtekio av. 11, Vilnius LT-10223, Lithuania

E-mail: `raimondas.ciegis@vgtu.lt`

We construct and investigate efficient parallel finite volume schemes for solution of partially dimension reduced nonlocal heat conduction problems. Let us restrict to 2D space domains. In the case of classical diffusion operators the following hybrid dimension PDE model is defined in [1]:

$$\begin{aligned}
&\frac{\partial U}{\partial t} + AU = 0, \quad (x,y,t) \in (0,\delta) \cup (X-\delta, X) \times (0,Y) \times (0,T], && (1)\\
&\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \quad (x,t) \in (\delta, X-\delta) \times (0,T], && (2)\\
&U(0,y,t) = g_1(y,t), \;\; U(X,y,t) = g_2(y,t), \quad (y,t) \in [0,Y] \times (0,T], \\
&\frac{\partial U}{\partial y}(x,0,t) = 0, \quad \frac{\partial U}{\partial y}(x,Y,t) = 0, \qquad (x,t) \in (0,\delta) \cup (X-\delta, X) \times (0,T].
\end{aligned}$$

The following two pairs of conjugation conditions are valid at the truncation lines:

$$\begin{aligned}
&U\big|_{x_\delta - 0} = U\big|_{x=\delta+0}, \quad U\big|_{x=X-\delta-0} = U\big|_{x=X-\delta+0}, \\
&\frac{\partial S(U)}{\partial x}\Big|_{x_\delta-0} = \frac{\partial U}{\partial x}\Big|_{x=\delta+0}, \quad \frac{\partial U}{\partial x}\Big|_{x=X-\delta-0} = \frac{\partial S(U)}{\partial x}\Big|_{x=X-\delta+0}. && (3)
\end{aligned}$$

Our main aim is to generalize this technique for the nonlocal fractional power diffusion operators A^α, where $0 < \alpha < 1$. The spectral definition of such operators is used in our analysis.

Numerical finite volume schemes for solution of parabolic problems with fractional power elliptic operators are investigated in [2]. Here a special theoretical framework is proposed to split the full convergence analysis into three separate parts. For any new problem it is sufficient to analyze only specific parts of this problem.

REFERENCES

[1] A. Amosov, G. Panasenko. Partial dimension reduction for the heat equation in a domain containing thin tubes. *Mathematical Methods in the Applied Sciences*, **41** (18):9529–9545, 2018.

[2] R. Čiegis, I. Dapšys. On a framework for the stability and convergence analysis of discrete schemes for nonstationary nonlocal problems of parabolic type. *Mathematics*, **10** (4): 2155–2167, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

1D CONVOLUTIONAL NEURAL NETWORKS FOR SOLUTION OF THE BIOSENSOR INVERSE PROBLEM

IGNAS DAPŠYS, VADIMAS STARIKOVIČIUS

Vilnius Gediminas Technical University

Saulėtekio ave. 11, LT-10223, Vilnius, Lithuania

E-mail: ignas.dapsys@vilniustech.lt, vadimas.starikovicius@vilniustech.lt

Biosensors are devices for the detection and analysis of chemical compounds based on biochemical processes. Since real-world experiments involving biosensors are expensive and time consuming, mathematical modelling is widely used in their development [1]. Our main interest is in studying the inverse biosensor problem – given a biosensor signal for an unknown sample, determine its composition. This requires solving the forward model equations for signals, whose composition is known, to establish an inverse dependence. Once this is done, unknown samples can be analyzed. But, unless we have a biosensor for a single substrate, this process is less straightforward, since for multiple substrates this problem is ill-posed [2]. This may lead to the deterioration of biosensor precision, especially when the biosensor response is under the effect of noise [3].

One promising method to alleviate the ill-posedness of such problems is to use neural networks, which have successfully been applied to solve the inverse biosensor problem [4]. Here, the authors use the feedforward neural network architecture, however, this raises the question whether such a simple architecture is optimal for this particular use case when the biosensor signals are corrupted by noise. In this presentation, we propose to use one-dimensional convolutional neural networks for the biosensor problem. This architecture is employed in spectroscopy [5], where obtained spectra are often noisy, due to its ability to remove noise [6]. We present our findings on biosensor precision for noisy signals, and the effect of measurement domain shrinkage for this architecture.

REFERENCES

[1] R. Baronas, F. Ivanauskas, J. Kulys. *Mathematical Modeling of Biosensors*. Springer, 2021.

[2] A. Žilinskas, D. Baronas. Optimization-based evaluation of concentrations in modeling the biosensor-aided measurement. *Informatica*, **22** (4):589–600, 2011.

[3] R. Baronas, J. Kulys, A. Lančinskas, A. Žilinskas. Effect of diffusion limitations on multianalyte determination from biased biosensor response. *Sensors*, **14** (3):4634–4656, 2014.

[4] L. Litvinas, R. Baronas. Application of artificial neural networks and biosensors to determine concentrations of mixture. *Lietuvos matematikos rinkinys (LMD darbai)*, **55** (B):78–83, 2014. (in Lithuanian)

[5] M. H. Mozaffari, L.L. Tay. A review of 1D convolutional neural networks toward unknown substance identification in portable Raman spectrometer. *arXiv preprint arXiv:2006.10575*, 2020.

[6] S. Pakravan, P.A. Mistani, M.A. Aragon-Calvo, F. Gibou. Solving inverse-PDE problems with physics-aware neural networks. *Journal of Computational Physics*, **440** :110414, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

A STUDY OF THERMAL DECOMPOSITION OF POROUS BIOMASS

MARIS GUNARS DZENIS

Department of Mathematics, University of Latvia

Jelgavas street 3, Riga, Latvia

E-mail: maris_gunars.dzenis@lu.lv

This paper deals with theory and simulations for mathematical model of biomass thermal decomposition. Objective of the study is to simulate pyrolysis of straw, wood and peat mixtures with different microwave pretreatments. Biomass thermal decomposes as a volatile part and carbon part which reacts with oxygen creating additional heat. Biomass is viewed as a porous medium. Chemical reactions are modeled according to Arrhenius kinetics. Biomass is described by components lignin, cellulose and hemicellulose for the purpose to model the thermal decomposition difference between straw, wood and peat. Gases in this model is viewed like an ideal gas which covered by the Darcy law and mass balance equation [1]. Porosity changes within process of biomass thermal decomposition. Numerical solutions were found using upwind and finite volume methods in program MatLab.

REFERENCES

[1] U. Strautins, L. Leja, M.G. Dzenis. Some network models related to heat and mass transfer during thermal conversion of biomass. *Enginnering for rural development*, **20** :1213–1218, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

UNIVERSALITY OF CERTAIN COMPOSITIONS

VIRGINIJA GARBALIAUSKIENĖ[1], MONIKA TEKORĖ[2]

[1]*Šiauliai Academy, Vilnius University*

P. Višinskio street 25, LT-76351 Šiauliai, Lithuania

E-mail: `virginija.garbaliauskiene@sa.vu.lt`

[2]*Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: `monika.tekore@mif.stud.vu.lt`

Let $s = \sigma + it$ be a complex variable, and $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ a periodic sequence of complex numbers. The periodic zeta-function $\zeta(s;\mathfrak{a})$, for $\sigma > 1$, is given by the Dirichlet series $\zeta(s;\mathfrak{a}) = \sum_{m=1}^{\infty} a_m m^{-s}$, and by analytic continuation elsewhere. If the coefficients are multiplicative, then the function $\zeta(s;\mathfrak{a})$ is universal [1], i. e., its shifts $\zeta(s + i\tau;\mathfrak{a})$ approximate a class of analytic functions defined on the strip $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$. In [2] and [3] joint universality theorems on approximation of collection of analytic functions by shifts $(\zeta(s + i\gamma_1(\tau);\mathfrak{a}_1), \dots, \zeta(s + i\gamma_r(\tau);\mathfrak{a}_r))$ and $(\zeta(s+ih_1\gamma_1;\mathfrak{a}_1), \dots, \zeta(s+ih_r\gamma_r;\mathfrak{a}_r))$, respectively, were proved. Here $\gamma_1(\tau), \dots, \gamma_r(\tau)$ are certain differentiable functions, and $\{\gamma_k : k \in \mathbb{N}\}$ is a sequence of positive imaginary parts of non-trivial zeros of the Riemann zeta-function.

Let $H(D)$ be the space of analytic on D functions. In the report, we consider the universality of the functions $F(\zeta(s;\mathfrak{a}_1), \dots, \zeta(s;\mathfrak{a}_r))$, where $F : H^r(D) \to H(D)$ is a certain operator. We give one example. Denote by $\mathcal{K}$ the class of compact sets of D, and by $H(K)$, $K \in \mathcal{K}$, the class of continuous functions on K that are analytic in the interior of K, and let $S = \{g \in H(D) : g(s) \neq 0 \text{ or } g(s) \equiv 0\}$.

THEOREM 1. *Suppose that the sequences $\mathfrak{a}_1, \dots, \mathfrak{a}_r$ are multiplicative, $h_1, \dots, h_r$ are positive algebraic numbers linearly independent over $\mathbb{Q}$, and estimate $\sum_{(\gamma_k,\gamma_l):|\gamma_k-\gamma_l|<1/\log T} 1 \ll T \log T$, $T \to \infty$, is valid. Let $F : H^r(D) \to H(D)$ be a continuous operator, such that $\{g \in H(D) : g(s) \neq c \in \mathbb{C}\} \subset F(S^r)$. Let $K \in \mathcal{K}$, $f(s) \in H(K)$ and $f(s) \neq c$ on K. Then, for every $\varepsilon > 0$,*

$$\liminf_{N\to\infty} \frac{1}{N} \# \left\{ 1 \leqslant k \leqslant N : \sup_{s \in K} |F(\zeta(s + ih_1\gamma_k;\mathfrak{a}_1), \dots \zeta(s + ih_r\gamma_k;\mathfrak{a}_r)) - f(s)| < \varepsilon \right\} > 0.$$

For example, we can take $F(g_1, \dots, g_r) = b_1 g_1 + \dots + b_r g_r$, $g_1, \dots, g_r \in H(D)$, $b_1, \dots, b_r \in \mathbb{C} \setminus \{0\}$.

REFERENCES

[1] A. Laurinčikas, D. Šiaučiūnas. Remarks on the universality of the periodic zeta-function. *Mathematical Notes*, **80** (1-2):532–538, 2006.

[2] A. Laurinčikas, M. Tekorė. Joint universality of periodic zeta-functions with multiplicative coefficients. *Nonlinear Analysis: Modelling and Control*, **25** (5):860–883, 2020.

[3] A. Laurinčikas, D. Šiaučiūnas, M. Tekorė. Joint universality of periodic zeta-functions with multiplicative coefficients. II. *Nonlinear Analysis: Modelling and Control*, **26** (3):550–564, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

STOCHASTIC MODELING OF ANIMAL POPULATION DYNAMICS

JOLANTA GOLDŠTEINE, ANDREJS MATVEJEVS, OKSANA PAVLENKO

Riga Technical University

Zunda embankment 10, Riga LV-1007, Latvia

E-mail: jolanta.goldsteine@gmail.com,andrejs.matvejevs@rtu.lv,oksana.pavlenko@rtu.lv

The most popular classical mathematical model for biological pest control may be given as a system of ordinary differential equations (see, for example, in A. D. Bazykin at all [1] and Yu. M. Svirezhef and D. I. Logofet [2] system of the equations (1)):

$$\begin{cases} \frac{dx}{dt} = xf(x) - g(x,y), \\ \frac{dy}{dt} = yh(y) + m(x,y), \end{cases} \tag{1}$$

where x is a density of a pest (or prey) population and y is a density of a predator population, functions $f(x)$ and $h(y)$ define the relative growth rates of populations without contacts, functions $g(x,y)$ and $m(x,y)$ are changes of populations, growth rates caused by a cooperative effect, called functional responses. In reality, even through all functions of the right part in (1) are derived in compliance with biological laws, the parameters of these functions are random and should be estimated by collecting and analyzing environmental data. Therefore, several papers propose and analyze mathematical models for interacting populations in a form of stochastic Ito differential equations (see, for example, the book of J. Murray [3] and references there):

$$\begin{cases} dx(t) = \big[x(t)f\big(x(t)\big) - g\big(x(t),y(t)\big)\big]dt + \sigma_1\big(x(t),y(t)\big)dw_1(t), \\ dy(t) = \big[y(t)h\big(y(t)\big) + m\big(x(t),y(t)\big)\big]dt + \sigma_2\big(x(t),y(t)\big)dw_2(t), \end{cases} \tag{2}$$

where $w_1(t)$ and $w_2(t)$ are correlated Wiener processes given on probability space $(\Omega, \mathfrak{F}, \mathbb{P})$.

The paper deals with population mathematical models given in a form of fast oscillating stochastic impulsive differential equations. The proposed approximative algorithm for quantitative analysis of population dynamics consists of two steps. First, we construct an ordinary differential equation for the mean value of population growth and analyse the average asymptotic population behaviour. Then, applying diffusion approximation procedure, we derive the stochastic Ito differential equation for random deviations on the above average motion and analyse probabilistic characteristics of possible stationary population state. The algorithm is applied to the model of biological pest control of Holling type II.

REFERENCES

[1] A.D. Bazykin, A.I. Rhibnik, B. Krauskopf. *Nonlinear Dynamics of Interacting Populations.* World Scientific, Singapore, 1998.

[2] Yu.M. Svirezhev, D.J. Logofet. *Stability of Biological Communities.* Moscow, MIR Publishers, 1983.

[3] J. Murray. *Mathematical Biology I: An Introduction.* Volume I. Springer-Verlag, 3rd edition, 2003.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

AGGREGATION BASED FUZZY RELATIONS IN ANTI-MONEY LAUNDERING MONITORING ALGORITHM

OLGA GRIGORENKO

Institute of Mathematics and Computer Science, University of Latvia

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `ol.grigorenko@gmail.com`

Anti-Money Laundering (AML) is a set of laws and regulations designed to prevent the illegal concealment of money obtained through criminal activity. Anti-Money Laundering typically involves analyzing large amounts of financial transaction data to identify patterns or behaviors that may indicate money laundering or other illicit financial activities. Various mathematical algorithms and techniques, mostly machine learning algorithms, can be used in AML to help detect and prevent such activities. However, in AML, the data sets are often highly imbalanced, with a small proportion of transactions being suspicious or fraudulent. This presents a challenge for machine learning algorithms, as they rely on having a large amount of data to learn from and make accurate predictions. When the number of bad transactions is small, what usually happens in real life, it becomes difficult for machine learning algorithms to identify patterns or features that distinguish them from the vast majority of legitimate transactions. The other tools which is used for ALM is rule-based algorithms – a type of approach that uses pre-defined rules to detect potentially suspicious financial transactions. These rules are based on specific criteria that are known for decition maker. Rule-based algorithms can also be implemented using fuzzy techniques and aggregation functions [2]. Overall, rule-based algorithms can be a useful tool in an AML program, but they should be used in conjunction with other methods such as machine learning and expert analysis to achieve the best results.

In our work we propose an alernative approach to existing ones, where rule-based algorithm is presenting by an aggregation function [1] and the degree of similarity to a suspicious transaction is determined by a fuzzy equivalence relation [3]. Thus, we build a aggregation based fuzzy equivalence relation, which is going to allow us identify the degree to which a transaction is illicit one.

Acknowledgment. The author is thankful for financial support European Regional Development Fund within the project Nr.1.1.1.2/16/I/001, application Nr.1.1.1.2/VIAA/4/20/707 “Fuzzy relations and fuzzy metrics for customer behavior modeling and analysis”.

REFERENCES

[1] M. Grabisch, J.-L. Marichal, E. Pap. *Aggregation Functions (Encyclopedia of Mathematics and its Applications).* Cambridge University Press, UK, 2009.

[2] M. Krastins. Fuzzy approach based money laundering risk assessment. In: *Proceedings of the 11th Conference of the European Society for Fuzzy Logic and Technology* , EUSFLAT, 2019.

[3] L.A. Zadeh. Similarity relations and fuzzy orderings. *Inform. Sci.*, **3** :170–200, 1971.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

LINEAR STABILITY OF A COMBINED CONVECTIVE FLOW IN AN ANNULUS

ARMANDS GRITSANS[1], VALENTINA KOLISKINA[2], ANDREI KOLYSHKIN[2], FELIX SADYRBAEV[3]

[1] *Daugavpils University,* [2] *Riga Technical University,* [3] *University of Latvia*

Zunda embankment 10, Riga LV-1048, Latvia

E-mail: `armands.gricans@du.lv, valentina.koliskina@rtu.lv, andrejs.koliskins@rtu.lv,`
E-mail: `felix@latnet.lv`

We consider convective flow in a tall vertical annulus. The annulus is assumed to be closed so that the total fluid flux through the cross-section of the channel is equal to zero. Steady convective flow in the vertical direction is generated due to two factors: (a) different constant temperatures of the walls and (b) internal heat generation due to chemical reaction that takes place in the fluid [1]. The problem is characterized by three dimensionless parameters: the Grasshof number Gr, the Frank-Kamenetskii parameter F characterizing heat release due to chemical reaction and the Prandtl number Pr.

The boundary value problem for steady state temperature distribution is nonlinear. Bifurcation analysis performed in the paper (similarly to the approach used [2]) shows that for each $R = R_1/R_2$, where R_1 and R_2 are the inner and outer radii of the annulus, respectively, there are two solutions in the interval $0 < F < F^*$, where F^* depends on R. The solution with the smallest norm is physically realizable and should be selected for linear stability analysis.

Linear stability problem is solved numerically for two cases considered in the paper: Case 1 (different constant wall temperatures) and Case 2 (zero wall temperatures). In Case 1, multiple minima on the marginal stability curves are obtained as the Prandtl number increases. The global minimum is selected among two or three local minima associated with different wave numbers and depends on the particular value of the Prandtl number. As a result, finite jumps in the value of the wave number can be observed for different Pr leading to the changes in the form of the most unstable perturbation. The critical Grashof numbers for Case 1 do not change much until the transition to the mode with different wave number takes place, after that the critical Gr decreases rapidly. Calculations show that for Case 2 marginal stability curves have one minimum and the critical Grashof numbers decrease monotonically as the Prandtl number grows. In both cases (Case 1 and Case 2) the increase in F leads to less stable flow.

Acknowledgment. This research was funded by the Latvian Council of Science project No. lzp-2020/1-0076.

REFERENCES

[1] D.A. Frank-Kamenetskii. *Diffusion and Heat Exchange in Chemical Kinetics.* Princeton University Press, Princeton, 1955.

[2] J. Bebernes, D. Eberly. *Mathematical Problems from Combustion Theory.* Springer-Verlag, New York, 1989.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

MULTIPLICITY RESULTS FOR BOUNDARY VALUE PROBLEMS WITH THE ARRHENIUS NONLINEARITY

ARMANDS GRITSANS[1], ANDREI KOLYSHKIN[2], DIANA OGORELOVA[1], FELIX SADYRBAEV[1], INNA SAMUILIK[1,2], INARA YERMACHENKO[1]

[1] *Daugavpils University*

Parades 1, LV-5401, Daugavpils, Latvia

[2] *Riga Technical University*

Daugavgrivas 2-135, LV-1048, Riga, Latvia

E-mail: `armands.gricans@du.lv, andrejs.koliskins@rtu.lv, diana.ogorelova@inbox.lv`

E-mail: `felix@latnet.lv, lvinna@inbox.lv, inara.jermacenko@du.lv`

Various models of heat conduction uses the Arrhenius nonlinearity

$$f(T) = e^{-\frac{\theta}{T}}, \quad \theta > 0,$$

representing the temperature distribution.

We consider the following boundary value problems:

$$T'' + \frac{T'}{r} + F\, e^{-\frac{\theta}{T}} = 0, \qquad T(R) = 0 = T(1); \tag{1}$$

$$T'' + \frac{T'}{r} + F\, e^{-\frac{\theta}{T}} = 0, \qquad T(R) = T_f = T(1); \tag{2}$$

$$Q'' + \frac{Q'}{r} + F e^{-\frac{\theta}{Q+T_f}} = 0, \qquad Q(R) = 0 = Q(1), \tag{3}$$

where θ, F, T_f are positive numbers and $R \in (0, 1)$.

We are interested in positive solutions of problems (1)-(3).

Bifurcation analysis of these problems with respect to the parameters θ, F, T_f, and R allows us to obtain some results on the existence and number of positive solutions of (1)-(3).

Acknowledgment. This research is funded by the Latvian Council of Science, project "Analysis of complex dynamical systems in fluid mechanics and heat transfer", project No. lzp-2020/1-0076.

REFERENCES

[1] J. Bebernes, D. Eberly. *Mathematical Problems from Combustion Theory.* Springer-Verlag, New York, 1989.

[2] J.D. Buckmaster, G.S.S. Ludford. *Lectures on Mathematical Combustion.* CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, 1983.

[3] P. Korman. *Global Solution Curves for Semilinear Elliptic Equations.* World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2012.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

TAMING COMPLEXITY WITH OPTIMAL CONTROL: FROM ECOLOGY TO EPIDEMIOLOGY

DMITRY GROMOV

Department of Mathematics, University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: gromov@lu.lv

Most real-life problems are characterized by their inherent complexity. Ecological systems may undergo abrupt changes in their structure and functioning, which are typically referred to as regime switches. Once the regime shift happened, the system cannot return to its previous state, until another threshold is crossed. This results in the occurrence of multi-stability and hysteresis-like behavior, which is observed in the populations of insects and further natural phenomena, see [3]. When we turn to the epidemiological applications, their inherent complexity is caused not only by the highly non-linear dynamics, but also by a complicated interplay of structural and budgetary restrictions. This complexity substantially restricts our ability to determine the strategies aimed at combatting the spread of the disease.

In this talk, we will argue that optimal control theory offers a comprehensive framework for dealing with such problems. Due to its flexibility and a wide range of developed methods, it can address a broad spectrum of real problems. To illustrate this thesis, we will consider the application of hybrid optimal control, particularly the hybrid Pontryagin maximum principle, [1], to the problem of pollution control with periodic regime shifts, [4]. Another application comes from the realm of numerical optimal control and demonstrates the computation of the optimal treatment and prophylaxis investment profiles for the problem of reducing the incidence of HIV while obeying complex budgetary restrictions, [2].

The talk will be concluded by a brief overview of open problems and future directions of research.

REFERENCES

[1] V. Azhmyakov, S.A. Attia, D. Gromov, J. Raisch. Necessary optimality conditions for a class of hybrid optimal control problems. In: *Proc. of the 10th International Workshop, HSCC 2007, Pisa, Italy, April 3-5, 2007*, Hybrid Systems: Computation and Control, Bemporad, A., Bicchi, A., Buttazzo, G. (Eds.), Springer, 637–640, 2007.

[2] D. Gromov, I. Bulla, O.S. Serea, E.O. Romero-Severson. Numerical optimal control for HIV prevention with dynamic budget allocation. *Mathematical Medicine and Biology: A Journal of the IMA*, **35** (4):469–491, 2018.

[3] D. Gromov, T. Upmann. Dynamics and economics of shallow lakes: A survey. *Sustainability*, **13** (24):13763, 2021.

[4] D. Gromov, T. Shigoka, A. Bondarev. Optimality and sustainability of hybrid limit cycles in the pollution control problem with regime shifts. *Environment, Development and Sustainability*, 1–18, 2023. (online first)

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON HEURISTIC PARAMETER CHOICE FOR THE CLASS OF REGULARIZATION METHODS

UNO HÄMARIK, TOOMAS RAUS

University of Tartu

Narva mnt 18, Tartu, 51009, Estonia

E-mail: uno.hamarik@ut.ee, toomas.raus@ut.ee

Let H and F be Hilbert spaces and $A \in L(H,F)$. We consider the ill-posed operator equation

$$Au = f, \quad f \in \mathcal{R}(A) \neq \overline{\mathcal{R}(A)}. \tag{1}$$

The kernel $N(A)$ may be non-trivial. Instead of an exact right-hand side f_* we have only an approximation $f \in F$. For the regularization of problem (1) we consider the following class of regularization methods:

$$u_r = (I - A^*Ag_r(A^*A))u_0 + g_r(A^*A)A^*f.$$

Here u_0 is the initial approximation, r is the regularization parameter, I is the identity operator and the generating function $g_r(\lambda)$ satisfies for $r \geq 0$ the conditions

$$\sup_{0\leq\lambda\leq\|A^*A\|} \left|g_r(\lambda)\right| \leq \gamma r, \qquad \sup_{0\leq\lambda\leq\|A^*A\|} \lambda^p \left|1-\lambda g_r(\lambda)\right| \leq \gamma_p r^{-p}, \quad 0 \leq p \leq p_0.$$

Examples of methods of this class are (iterated) Tikhonov method, Landweber iteration method, implicit iteration method, method of asymptotical regularization, the truncated singular value decomposition methods etc.

If the noise level of data is unknown, for the choice of the regularization parameter r heuristic rule is needed. We propose to choose r from the set L_{min} of the local minimum points of the quasioptimality criterion function

$$\psi_Q(r) = r\|A^*(I - AA^*g_r(AA^*))^{\frac{2}{p_0}}(Au_r - f)\|$$

on the set of parameters $\Omega = \left\{r_j : r_j = qr_{j-1},\, j = 1,2,...,M,\, q > 1\right\}$. Then the following error estimates hold:

a)

$$\min_{r\in L_{min}} \|u_r - u_*\| \leq C \min_{r_0\leq r\leq r_M} \left\{\left\|u_r^+ - u_*\right\| + \left\|u_r - u_r^+\right\|\right\}.$$

Here u_* and u_r^+ are the exact and regularized solutions of equation $Au = f_*$ and the constant $C \leq c_q \ln(r_M/r_0)$ can be computed for each individual problem $Au = f$.

b) Let $u_* = (A^*A)^{p/2}v$, $\|v\| \leq \rho$. If $r_0 = 1$, $r_M = c\,\|f - f_*\|^{-2}$, $c = (2\,\|u_*\|)^2$, then

$$\min_{r\in L_{min}} \|u_r - u_*\| \leq c_{p,q}\rho^{1/(p+1)} \left|\ln\|f - f_*\|\right| \|f - f_*\|^{p/(p+1)}, 0 < p \leq 2p_0.$$

We consider also some algorithms for parameter choice from the set L_{min}.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON THE MISHOU THEOREM FOR PERIODIC ZETA-FUNCTIONS

MINDAUGAS JASAS, RENATA MACAITIENĖ

Institute of Mathematics, Faculty of Mathematics and Informatics, Vilnius University

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: mindaugas.jasas@mif.stud.vu.lt, renata.macaitiene@sa.vu.lt

Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ and $\mathfrak{b} = \{b_m : m \in \mathbb{N}_0\}$ be two periodic sequences of complex numbers, $0 < \alpha \leqslant 1$ be a fixed number, and $s = \sigma + it$. The periodic and periodic Hurwitz zeta-functions are defined, for $\sigma > 1$, by the series $\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s}$, $\zeta(s, \alpha; \mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b_m}{(m+\alpha)^s}$, and have meromorphic continuations to the whole complex plane.

In this talk, we will consider the joint approximation of analytic functions by shifts of some absolutely convergent Dirichlet series connected to the functions $\zeta(s; \mathfrak{a})$ and $\zeta(s, \alpha; \mathfrak{b})$. Our result extends the well-known Mishou theorem for the Riemann and Hurwitz zeta-functions [1].

Denote by $v_u(m) = \exp\{-(m/u)^\theta\}$, $m \in \mathbb{N}$, and $v_u(m, \alpha) = \exp\{-((m+\alpha)/u)^\theta\}$, $m \in \mathbb{N}_0$, with $u > 0$ and fixed $\theta > \frac{1}{2}$, and define the series $\zeta_u(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m v_u(m)}{m^s}$ and $\zeta_u(s, \alpha; \mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b_m v_u(m,\alpha)}{(m+\alpha)^s}$, which are absolutely convergent for $\sigma > 1$. We will consider the approximation of analytic functions by shifts $(\zeta_{u_T}(s + i\tau; \mathfrak{a}), \zeta_{u_T}(s + i\tau, \alpha; \mathfrak{b}))$, where $u_T \to \infty$ as $T \to \infty$. Now, define two tori $\Omega_1 = \prod_p \gamma_p$ and $\Omega_2 = \prod_{m \in \mathbb{N}_0} \gamma_m$, where γ_p and γ_m are the unit circles (for all $p \in \mathbb{P}$ and $m \in \mathbb{N}_0$), and let $\Omega = \Omega_1 \times \Omega_2$. Thus, on $(\Omega, \mathcal{B}(\Omega))$, the probability Haar measure m_H can be defined. Denote by $\omega_1(p)$ the pth component of an element $\omega_1 \in \Omega_1$, and by $\omega_2(m)$ the mth component of an element $\omega_2 \in \Omega_2$, and construct two random elements $\zeta(s, \omega_1; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m \omega_1(m)}{m^s}$ and $\zeta(s, \alpha, \omega_2; \mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b_m \omega_2(m)}{(m+\alpha)^s}$, where the function $\omega_1(m)$ is the extension of $\omega_1(p)$ to the set $\mathbb{N}$. Note that the latter series are uniformly convergent on compact subsets of the strip $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$ for almost all ω_1 and ω_2. Moreover, let $\mathcal{K}$ be the class of compact subsets of D with connected complements, $H(K)$ (with $K \in \mathcal{K}$) the class of continuous functions on K that are analytic in the interior of K, and $H_0(K)$ is the subclass of $H(K)$ consisting of non-vanishing functions.

THEOREM 1. *Suppose that the sequence $\mathfrak{a}$ is multiplicative, α is transcendental, $u_T \to \infty$ and $u_T \ll T^2$ as $T \to \infty$. Let $K_1, K_2 \in \mathcal{K}$ and $f_1(s) \in H_0(K)$, $f_2(s) \in H(K)$. Then the limit*

$$\lim_{T\to\infty} \frac{1}{T} \operatorname{meas} \left\{ \tau \in [0, T] : \sup_{s \in K_1} |\zeta_{u_T}(s + i\tau; \mathfrak{a}) - f_1(s)| < \varepsilon_1, \sup_{s \in K_2} |\zeta_{u_T}(s + i\tau, \alpha; \mathfrak{b}) - f_2(s)| < \varepsilon_2 \right\}$$
$$= m_H \left\{ (\omega_1, \omega_2) \in \Omega : \sup_{s \in K_1} |\zeta(s, \omega_1; \mathfrak{a}) - f_1(s)| < \varepsilon_1, \sup_{s \in K_2} |\zeta(s, \alpha, \omega_2; \mathfrak{b}) - f_2(s)| < \varepsilon_2 \right\} > 0$$

exists for all but at most countably many $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$.

REFERENCES

[1] H. Mishou. The joint value-distribution of the Riemann zeta-function and Hurwitz zeta-functions. *Lith. Math. J.*, **47** (1):32–47, 2007.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

NAVIER-STOKES EQUATION IN A THIN TUBE STRUCTURE MOTIVATED BY HEMODYNAMICS

RITA JUODAGALVYTĖ

Vilnius University

Naugarduko st. 24, Vilnius LT-03225, Lithuania

E-mail: rita.juodagalvyte@mif.vu.lt

The steady-state Navier-Stokes equation in a thin tube structure B_ε, with the inflow-outflow boundary conditions for the Bernoulli pressure, is considered (see (1))

$$\begin{cases} -\nu\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p = 0, & x \in B_\varepsilon, \\ \operatorname{div}\mathbf{v} = 0, & x \in B_\varepsilon, \\ \mathbf{v} = 0, & x \in \partial B_\varepsilon \setminus \cup_{j=N_1+1}^{N} \gamma_\varepsilon^j, \\ \mathbf{v}_{\boldsymbol{\tau}} = 0, & x \in \gamma_\varepsilon^j, \\ -\nu\partial_n(\mathbf{v}\cdot\mathbf{n}) + \left(p + \frac{1}{2}|\mathbf{v}|^2\right) = c_j/\varepsilon^2, & x \in \gamma_\varepsilon^j,\ j = N_1+1, ..., N, \end{cases} \tag{1}$$

where $\mathbf{v}_{\boldsymbol{\tau}} = \mathbf{v} - (\mathbf{v}\cdot\mathbf{n})\mathbf{n}$ is the tangential component of the vector $\mathbf{v}$, $\partial_n h = \nabla h \cdot \mathbf{n}$ is the normal derivative of h, c_j are some constants. The existence and uniqueness of the weak solution taking into account the domain dependence on the small parameter were proved. Also, the asymptotic expansion of a weak solution of the stationary Navier-Stokes equations in the whole thin tube structure with the given Bernoulli boundary conditions for the inflows and outflows was constructed. The constructed asymptotic expansion allowed reducing the computational costs. Indeed, full-dimensional computations are only needed in small neighborhoods of the junction of tubes, while in the main part of the domain, the computations are one-dimensional. Joint work with Grigory Panassenko and Konstantinas Pileckas.

REFERENCES

[1] R. Juodagalvytė, G. Panassenko, K. Pileckas. Steady-state Navier-Stokes equations in thin tube structure with the Bernoulli pressure inflow boundary conditions: Asymptotic analysis. *Mathematics*, **9** (19):1–20, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

REMARKS ON SIMULTANEOUS APPROXIMATION BY THE CLASSES OF ZETA-FUNCTIONS

ROMA KAČINSKAITĖ

Institute of Mathematics, Faculty of Mathematics and Informatics, Vilnius University

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: `roma.kacinskaite@mif.vu.lt`

The universality property in the Voronin sense of the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, says roughly speaking that any analytic non-vanishing function can be approximated uniformly on any compact subsets of the strip $\{s \in \mathbb{C} : 1/2 < \sigma < 1\}$ by shifts of $\zeta(s)$. The mixed joint universality gives a possibility to connect into one tuple two different types of zeta-functions (one of those zeta-functions has the Euler product expression over primes and the other does not) and to study an analogous problem on the approximation. In other words, we can show that any two target functions can be approximated simultaneously by the suitable vertical shifts of two different type zeta- or L-functions and that the set of such shifts has a positive lower density.

The rather general result can be shown for the broad classes of zeta-functions, particularly, if an approximating tuple is composed of the Matsumoto zeta-functions' class and the periodic Hurwitz zeta function.

In the talk, we will present three cases of the mixed joint discrete universality property of abovementioned functions. These results are obtained by the author, Kohji Matsumoto (Nagoya University, Japan) and Łukasz Pańkowski (A. Mickiewicz University, Poland) (see [1], [2], [3], [4]).

REFERENCES

[1] R. Kačinskaitė, K. Matsumoto. On mixed joint discrete universality for a class of zeta-functions. In: *Proc. of 6th Intern. Conf., Palanga, Lithuania, 2016*, Anal. Probab. Methods Number Theory, A. Dubickas et al. (Eds.), Vilnius University Publ. House, Vilnius, 51–66, 2017.

[2] R. Kačinskaitė, K. Matsumoto. On mixed joint discrete universality for a class of zeta-functions: a further generalization. *Math. Modell. Anal.*, **25** (4):569–583, 2020.

[3] R. Kačinskaitė, K. Matsumoto. The discrete case of the mixed joint universality for a class of certain partial zeta-functions. *Taiwanese J. Math.*, **25** (4):647–663, 2021.

[4] R. Kačinskaitė, K. Matsumoto, Ł. Pańkowski. On mixed joint discrete universality for a class of zeta-functions: one more case. *Taiwanese J. Math.*, doi 10.11650/tjm/220804. Advance Publication, 1–16, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON INDUCED BIPOLAR OWA OPERATORS

MARTIN KALINA[1], BILJANA MIHAILOVIĆ[2], MIRJANA ŠTRBOJA[3]

[1] *Slovak University of Technology*

Radlinského 11, 810 05 Bratislava, Slovakia

[2,3] *University of Novi Sad*

[2] Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia

[3] Trg Dositeja Obradovića 3, 21000 Novi Sad, Serbia

E-mail: martin.kalina@stuba.sk, lica@uns.ac.rs, mirjana.strboja@dmi.uns.ac.rs

Ordered weighted averages (OWA) introduced by Yager [5] have shown their usefulness in multicriteria decision-making, and also in other fields where ranking is important. Later, OWA were generalized to induced OWA (IOWA) by Yager [6]. Induced means that the ranking is done by an inducing vector rather than by the vector itself to be aggregated. Recently, several other generalizations of OWA were introduced and studied. Important for our purposes will be also bipolar ordered weighted averages (BIOWA) recently introduced by Mesiar et al. in [3,4] that are based on the Choquet-integral with respect to bi-capacities (see [1,2]).

The main aim of this presentation is to introduce and investigate a generalization of BIOWA operators, in particular, induced bipolar ordered weighted averages.

Acknowledgment. The first author acknowledges the support of the the Science and Technology Assistance Agency under contract No. APVV-18-0052, and of the project VEGA 2/0142/20, the second and the third author wish to acknowledge the financial support of the Ministry of Education, Science and Technological Development of the Republic of Serbia, the second author by Grant No. 451-03-47/2023-01/200156, the third author by Grant No. 451-03-47/2023-01/200125.

REFERENCES

[1] M. Grabisch, Ch. Labreuche. Bi-capacities I: definition, Möbius transform and intersection. *Fuzzy Sets and Systems*, **151** (2):211–236, 2005.

[2] M. Grabisch, Ch. Labreuche. Bi-capacities II: Choquet integral. *Fuzzy Sets and Systems*, **151** (2):236–236, 2005.

[3] R. Mesiar, L. Jin, A. Stupňanová. BIOWA operators. In: *M.-J. Lesot, et al. (Eds.), IPMU 2020*, CCIS, vol. 1238, 419–425, 2020.

[4] R. Mesiar, A. Stupňanová, L. Jin. Bipolar ordered weighted averages: BIOWA operators. *Fuzzy Sets and Systems*, **433** :108–121, 2022.

[5] R.R. Yager. On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man and Cybernetics*, **18** (1):183–190, 1988.

[6] R.R. Yager. Induced aggregation operators. *Fuzzy Sets and Systems*, **137** (1 spec.):59–69, 2003.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

MATHEMATICAL MODELLING ELECTRICALLY DRIVEN FREE SHEAR FLOWS IN A DUCT UNDER UNIFORM MAGNETIC FIELD

HARIJS KALIS[1], ILMARS KANGRO[2]

[1] *Institute of Mathematics and Computer Science, University of Latvia*
Raina blvd. 29, Riga LV-1459, Latvia
[2] *Institute of Engineering, Faculty of Engineering, Rezekne Academy of Technologies*
Atbrivosanas aleja 115, LV-4601, Latvia
E-mail: `harijs.kalis@lu.lv, ilmars.kangro@rta.lv`

We consider a mathematical model of two-dimensional electrically driven laminar free shear flows in a straight duct under action of an applied uniform homogeneous magnetic field. The mathematical approach is based on studies by J.C.R. Hunt, W.E. Williams (1968) and Y. Kolesnikov, H.Kalis (2021). The magnetohydrodynamic (MHD) flow is characterized by one non-dimensional parameter Hartman (Ha) number. The MHD process is consider in duct cross section:

$$\Omega = \{(x, y, z) : -L \le x \le L, 0 \le y \le C, -\infty \le z \le +\infty\}$$

with electrically non conducting walls. External magnetic field in the (x, y) plane is applied as $\mathbf{B} = \{B_0 cos(\phi_0), B_0 sin(\phi_0), 0\}$,where B_0 is constant value of the magnetic field induction in the direction under angle ϕ_0. An electric current is supplied to duct by two couples of linear electrodes $z \in (-\infty, +\infty)$ at the duct walls $y = 0$ and $y = C$ by $x = \pm a$ perpendicular to the magnetic field. At the walls $y = 0$, $y = C$ in the first electrode $x = -a$ the electric current densities $J_x = 0, J_y = +\infty$, in the second electrode $x = +a$ the electric current densities $J_x = 0, J_y = -\infty$. Therefore $J_y = \delta(x + a) - \delta(x - a)$. We will find the symmetric distribution of azimuthal velocity $U = U(x, y)$ and induced magnetic field $H = H(x, y)(J_x = \frac{\partial H}{\partial y}, J_y = -\frac{\partial H}{\partial x})$ by solving the following boundary value problem for partial differential equations (PDEs):

$$\begin{cases} \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + Ha\,(cos(\phi_0)\frac{\partial H}{\partial x} + sin(\phi_0)\frac{\partial H}{\partial y}) = 0, \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + Ha\,(cos(\phi_0)\frac{\partial U}{\partial x} + sin(\phi_0)\frac{\partial U}{\partial y}) = 0, \\ x \in (-L, L), y \in (0, C), U(\pm L, y) = H(\pm L, y) = U(x, 0) = U(x, C) = 0, \\ H(x, 0) = H_0(x), H(x, C) = \gamma H_0(x), \end{cases}$$

where the constant $\gamma = -1; 0; 1$ is depending on the direction for the electric current injection in electrodes, $H_0(x)$ is the following function:

$$0, (x \in (-L, -a), x \in (a, L)), -I_0, (x \in (-a, a)),$$

where I_0 characterize the dimensionless value of the electric current ($I_0 = 1$).

Using transformations
$S_\pm = H \pm U, U = \frac{S_+ - S_-}{2}, H = \frac{S_+ + S_-}{2}$, $S_\pm(x, y) = W_\pm(x, y)exp(- \pm \alpha(f_0(x, y)))$, $f_0(x, y) = xcos(\phi_0) + ysin(\phi_0), \alpha = Ha/2$, we have PDEs problem for functions $W_\pm(x, y)$ solved analytically with Fourier method and numerically with Matlab.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

FUZZY VOLTERRA INTEGRAL EQUATION OF THE SECOND KIND WITH WEAKLY SINGULAR KERNEL

URVE KANGRO, ZAHRA ALIJANI

University of Tartu

Narva mnt 18, Tartu, Estonia

E-mail: `urve.kangro@ut.ee`

Fuzzy numbers are used instead of real numbers when there is some uncertainty in the data. Fuzzy function is a map from a set of real numbers to the set of fuzzy numbers.

We consider a fuzzy Volterra integral equation of the second kind

$$u(t) = f(t) + \int_0^t K(t,s)u(s)ds, \quad t \in [0,T],$$

where $K\colon D_T \to \mathbb{R}$ is a given kernel with domain $D_T = \{(t,s)\colon 0 \le s < t \le T\}$, f is a given fuzzy function and u is an unknown fuzzy function. The case of smooth kernel, $K \in C^2(\overline{D})$, where K may change sign on some lines, is considered in [1]. In the weakly singular case the kernel K is assumed to be in $C^m(D_T)$ for some $m \in \mathbb{N}$ and satisfy

$$\left| \left(\frac{\partial}{\partial t}\right)^j \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial s}\right)^l K(t,s) \right| \le C_{K,m} \begin{cases} 1 & \text{if} \quad j+\alpha < 0, \\ 1 + |\log(t-s)| & \text{if} \quad j+\alpha = 0, \\ (t-s)^{-j-\alpha} & \text{if} \quad j+\alpha > 0, \end{cases}$$

where $j, l \in \mathbb{N}_0$, $j + l \le m$ and $\alpha < 1$.

We consider the case where K is weakly singular and may change sign in D_T on some vertical and/or horizontal lines. Results on the existence, uniqueness, smoothness and fuzziness of the solution are proved.

To solve the integral equation numerically we propose a piecewise spline collocation method with a graded mesh. By increasing the number of collocation points we show that the numerical solution exists and converges to the exact solution. We obtain exact convergence rates depending on smoothness of the solution and on the grading parameter of the mesh and give sufficient conditions for the fuzziness of the approximate solution. We also present some numerical examples.

REFERENCES

[1] Z. Alijani, U. Kangro. Collocation method for fuzzy Volterra integral equations of the second kind. *Math. Model. Anal.*, **25** (1):146–166, 2020.

[2] Z. Alijani, U. Kangro. On the smoothness of solution of fuzzy Volterra integral equation of the second kind with weakly singular kernels. *Numer. Funct. Anal. Optim.*, **42** (7):819–833, 2021.

[3] Z. Alijani, U. Kangro. Numerical solution of a linear fuzzy Volterra integral equation of the second kind with weakly singular kernels. *Soft Computing*, **26** 12009–12022, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

INVERSE PROBLEM FOR THE HEAT EQUATION WITH ENERGY CONDITION

KRISTINA KAULAKYTĖ

Vilnius University

Naugarduko street 24, Vilnius, Lithuania

E-mail: kristina.kaulakyte@mif.vu.lt

The talk considers the inverse problem for the non-stationary heat equation in a bounded simply connected domain with a specific over-determination condition. The existence of a very weak solution is proved. This is the first step for studying the solvability of the Stokes problem with the prescribed kinetic energy. The motivation for such problems comes from the recent paper of T. Buchmaster and V. Vicol (see [1]).

This is joint work with Konstantinas Pileckas.

REFERENCES

[1] T. Buckmaster, V. Vicol. Nonuniqueness of weak solutions to the Navier-Stokes equation. *Annals of Mathematics*, **189** (1):101 – 144, 2019.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

NUMERICAL ANALYSIS OF A FDM SOLUTION TO A FLUID-STRUCTURE INTERACTION PROBLEM

KRISTINA KAULAKYTĖ[1], NIKOLAJUS KOZULINAS[1], GRIGORY PANASENKO[1,2], KONSTANTINAS PILECKAS[1], VYTENIS ŠUMSKAS[1]

[1] *Vilnius University, Institute of Applied Mathematics*

Naugarduko 24, Vilnius LT-03225, Lithuania

E-mail: `kristina.kaulakyte@mif.vu.lt, nikolajus.kozulinas@mif.vu.lt`

E-mail: `konstantinas.pileckas@mif.vu.lt, vytenis.sumskas@mif.vu.lt`

[2] *University of Lyon, UJM, Institute Camille Jordan*

UMR CNRS 5208, 23 rue P.Michelon, 42023, Saint-Etienne, France

E-mail: `grigory.panasenko@univ-st-etienne.fr`

A fluid-structure interaction model based on [1] was derived together with team members and is briefly presented in this talk. It couples two media with different physical characteristics. For example, it can be used to model blood flow in a small human arteriole or motor oil flow in a long, elastic pipe. Among some important aspects of this model, the inclusion of elasticity and viscosity of both media is stressed out.

Ir particular, a problem modelling blood flow in a small elastic arteriole is the main concern of this talk. By analysing the speed of blood flow averaged over cross-sections of the vessel, we get a 4th order PDE of the following form:

$$c_1Q_{tt} + c_2Q_{xxtt} + c_3Q_t + c_4Q_{xxt} + c_5Q_{xxxx} + c_6Q_{xx} + c_7Q = 0,$$

here Q is the averaged speed and coefficients c_i include physical characteristics of the model, such as the radius of tube, elastic tube thickness, density of the elastic medium, etc.

The main focus here is on some numerical aspects of the solution to the presented PDE problem. Namely, two schemes of different orders of accuracy are presented to solve it numerically. Their accuracies in time and space are analysed. The stability of proposed numerical schemes is investigated. Results of some interesting numerical tests are discussed to confirm the accuracy of constructed solvers.

The model is implemented for a single pipe geometry and for a Y-shaped network of vessels. Numerical results are compared with the full Navier-Stokes simulations. Numerical experiments confirm that the proposed ansatz is effectively applicable for a broad class of problems.

REFERENCES

[1] G.P. Panasenko, R. Stavre. Three dimensional asymptotic analysis of an axisymmetric flow in a thin tube with thin stiff elastic wall. *J. Math. Fluid Mech.*, **22** (20): 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

STABILITY OF A STEADY CONVECTIVE FLOW IN A VERTICAL CHANNEL WITH PERMEABLE BOUNDARIES

VALENTINA KOLISKINA, ANDREI KOLYSHKIN, INTA VOLODKO

Riga Technical University

Zunda embankment 10, Riga LV-1048, Latvia

E-mail: `valentina.koliskina@rtu.lv, andrejs.koliskins@rtu.lv, inta.volodko@rtu.lv`

Consider a tall vertical channel formed by two parallel infinite vertical planes $x = \pm h$. Viscous incompressible chemically reacting fluid is located in the region between the planes. A steady convective flow in the vertical direction is induced by the two factors: (a) nonlinear heat sources (heat is released as a result of a chemical reaction in accordance with the Arrhenius' law [1]), and (b) temperature difference between the walls of the channel. It is assumed that there exists also a steady flow in the direction perpendicular to the planes through the walls of the channel, namely, $U|_{x=\pm h} = \pm U_0$, where U_0 is a constant.

In the present paper linear stability problem for combined steady flow consisting of two velocity components is formulated and solved numerically. Linear stability is investigated using the method of normal modes. The boundary value problem for the system of ordinary differential equations is discretized using Chebyshev collocation method. The nonlinear boundary value problem for the base flow is solved numerically using Matlab. Stability boundary depends on the relationship between four parameters chracterizing the problem: the Prandtl number, the Frank-Kamenetskii parameter, the Reynolds number and the Grashof number. It is shown that for large Prandtl numbers instability is associated with thermal running waves propagating downstream with sufficiently large phase velocity. Calculations show that depending on the values of the parameters there exist regions of stabilization and destabilization in the parameter space. In particular, cross-flow stabilizes the flow while internal heat generation and temperature difference between the walls of the channel has a destabilizing effect.

Acknowledgment. Research reported in this publication was supported by the Latvian Council of Science under project No. lzp-2020/1-0076.

REFERENCES

[1] K.J. Laidler. The development of the Arrhenius equation. *Journal of Chemical Education*, **61** :494–498, 1984.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

IN SEARCH OF DYNAMICAL CHAOS

OLGA KOZLOVSKA[1], FELIX SADYRBAEV[2]

[1]*Riga Technical University*
Kipsalas street 6a, Riga LV-1048, Latvia

[2]*Institute of Mathematics and Computer Science, University of Latvia*
Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `olga.kozlovska@rtu.lv, felix@latnet.lv`

Systems of ordinary differential equations of the form

$$X' = F(\mu(WX - \Theta)) - X \tag{1}$$

are considered. Here F is a sigmoidal vector function, W is quadratic matrix with constant entries, μ and Θ are vectors of parameters. First, the three dimensional system is considered with a critical point which has characteristic numbers of the form $\lambda_1 > 0$, $\lambda_{2,3} = \alpha \pm \mathbf{i}\beta$, $\alpha < 0$, $\beta \neq 0$, $\mathbf{i} = \sqrt{-1}$. Such a point, accordingly to [1], is called saddle-focus(2,1). Second, the three dimensional system is considered with a critical point which has characteristic numbers of the form $\lambda_1 < 0$, $\lambda_{2,3} = \alpha \pm \mathbf{i}\beta$, $\alpha > 0$, $\beta \neq 0$. Such a point, accordingly to [1], is called saddle-focus(1,2). These cases are interchangeable by the reversing the independent variable t. The four dimensional system of the form (1) is considered also, where a critical point has the characteristic numbers $\lambda_{1,2} = \alpha_1 \pm \mathbf{i}\beta_1$, $\lambda_{3,4} = \alpha_2 \pm \mathbf{i}\beta_2$, $\alpha_1\alpha_2 < 0$, $\beta_{1,2} \neq 0$. The critical point is called saddle-focus(2,2). The dynamics of such systems essentially depends on the so called *saddle value* σ, which for the four dimensional system is $\sigma = \alpha_1 + \alpha_2$.

REFERENCES

[1] S.V. Gonchenko, A.S. Gonchenko, A.O. Kazakov, A.D. Kozlov, Yu.V. Bakhanova. Mathematical theory of dynamical chaos and its applications: Review. Part 2. Spiral chaos of three-dimensional flows. *Reviews of actual problems of nonlinear dynamics. Izvestiya VUZov. PND.*, **27** (5):7–52, 2019.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

BLOOD VELOCITY COMPUTATION INSIDE OF A HUMAN HEART LEFT ATRIUM APPENDAGE

NIKOLAJUS KOZULINAS

Vilnius University

Naugarduko st., 24, LT-03225 Vilnius, Lithuania

E-mail: nikolajus.kozulinas@mif.vu.lt

The talk is devoted to the numerical study of the Navier-Stokes equations in patient-specific geometry of the human heart left atrium. During atrial fibrillation, the left atrium and left atrium appendage (LAA) play a very important role in thrombogenesis. In the opinion of cardiologists, the reason of thrombogenesis is stagnation of blood flow in some parts of LAA, which likely depends on its geometry. Numerical solutions of the Navier-Stokes equations give a real patient blood flow velocity inside of LAA and detect the stagnation zones. The obtained results could help to prepare for surgical treatment and to make a conclusion about whether anticoagulants must be prescribed.

The results are obtained in collaboration with Sigita Aidietienė, Audrius Aidietis, Oleg Ardatov, Sergejus Borodinas, Rimgaudas Katkus, Kristina Kaulakytė, Grigory Panasenko, Konstantinas Pileckas.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

APPROXIMATION BY THE MELLIN TRANSFORM OF THE RIEMANN ZETA-FUNCTION

ANTANAS LAURINČIKAS

Faculty of Mathematics and Informatics, Vilnius University

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: antanas.laurincikas@mif.vu.lt

Let $\zeta(s)$, $s = \sigma + it$, denote the Riemann zeta-function. By the Voronin theorem the function $\zeta(s)$ is universal in the sense that its shifts $\zeta(s + i\tau)$, $\tau \in \mathbb{R}$, approximate a wide class of analytic functions.

In the theory of $\zeta(s)$, in particular, for investigations of the moments, the Mellin transforms

$$\mathcal{Z}_k(s) = \int_1^\infty \left| \zeta\left(\frac{1}{2} + ix\right) \right|^{2k} x^{-s}\,\mathrm{d}x, \quad k \in \mathbb{N},$$

are widely used. It turns out that the functions $\mathcal{Z}_k(s)$ also have good approximation properties. In the report, we consider the function $\mathcal{Z}(s) \stackrel{\mathrm{def}}{=} \mathcal{Z}_1(s)$.

Let $\delta > 0$ be a small fixed number, and $\Delta = \{s \in \mathbb{C} : 1/2 + \delta < \sigma < 1 - \delta\}$. Denote by $H(\Delta)$ the space of analytic on Δ functions endowed with the topology of uniform convergence on compacta. Our main result is the following statement.

THEOREM 1. *There exists a closed non-empty set $F \subset H(\Delta)$ such that, for every compact set $K \subset \Delta$, $f(s) \in F$, and every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas} \left\{ \tau \in [0, T] : \sup_{s\in K} |\mathcal{Z}(s + i\tau) - f(s)| < \varepsilon \right\} > 0.$$

Here meas$A$ denotes the Lebesgue measure of a measurable set $A \subset \mathbb{R}$. Unfortunately, the set F can't be given explicitly.

Theorem 1 is a consequence of a limit theorem in the space $H(\Delta)$ for the function $\mathcal{Z}(s)$. Let $\mathcal{B}(H(\Delta))$ denote the Borel σ-field of the space $H(\Delta)$.

THEOREM 2. *On $(H(\Delta), \mathcal{B}(H(\Delta)))$, there exists a probability measure P, such that*

$$\frac{1}{T} \mathrm{meas} \left\{ \tau \in [0, T] : \mathcal{Z}(s + i\tau) \in A \right\}, \quad A \in \mathcal{B}(H(\Delta)),$$

converges weakly to P as $T \to \infty$.

The functions $\mathcal{Z}_k(s)$ were introduced in [1].

REFERENCES

[1] Y. Motohashi. *Spectral Theory of the Riemann Zeta-Function.* Cambridge Univ. Press, Cambridge, 1997.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

STUDY OF TWO COMBINED HEAT, MASS TRANSFER AND REACTION MODELS WITH APPLICATIONS TO BIOMASS PELLET COMBUSTION

LAURA LEJA[1], ULDIS STRAUTIŅŠ[2]

[1] *Faculty of Physics, Mathematics and Optometry, University of Latvia*
Jelgavas street 3, Riga LV-1004, Latvia
[2] *Institute of Mathematics and Computer Science, University of Latvia*
Raina blvd. 29, Riga LV-1459, Latvia
E-mail: `ll13172@edu.lu.lv` and `uldis.strautins@lu.lv`

Extensive research has been devoted to simulating gasification or combustion processes [1],[2] in isotropic heterogeneous materials such as wood, wood briquettes, or pellets. Biomass pellets composed of wood, straw, peat, etc are a promising alternative to fossil fuels in many applications such as home heating. In order to further increase the efficiency while reducing harmful emissions, smart control methods have been proposed, e.g. using external electric and magnetic fields. If heat and mass transfer processes within the pellets are to be taken into account and the controlling device is to be of low computational power, then minimalistic mathematical models are required.

A network model has been previously proposed by the authors [3] however, the results have not been validated against more complete models. We develop an alternative model preserving the topology of the network model, consisting of nodes and channels connecting the nodes, with one-dimensional gas dynamics equations governing the gas flow between the nodes. The main object of research in the developed model is the development of a thermal gasification dynamics model, where the primary task is to simplify a real 3-dimensional model to a 1-dimensional model by discretizing pressure and flow with the help of a graph model validation and simulation. Mass conservation laws are used at the nodes to couple the gas dynamics equations on the various nodes. The resistance to gas flow between the nodes in the two models is described by different parameters permeability coefficient for the simple network model and channel length and diameter for the alternative model; these can be adjusted to apply both methods to a certain problem. The results of the two models are compared for some representative geometries.

REFERENCES

[1] M. Momeni, Ch. Yin, S.K. Kær, S.L. Hvid. Comprehensive study of ignition and combustion of single wooden particles. *Energy fuels*, **27** :1061 – 1072, 2013.

[2] Y.B. Yang, Ch. Ryu, A. Khor, V.N. Sharifi, J. Swithenbank. Fuel size effect on pinewood combustion in a packed bed. *Fuel*, **84** :2026 – 2038, 2005.

[3] L. Leja, U. Strautins. Network model for thermal conversion of heterogeneous biomass granules. In: *20st International Scientific Conference, Jelgava, 2021*, Engineering for Rural Development, 20.:737–743, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

A METHOD BASED ON CENTRAL PART INTERPOLATION FOR FRACTIONAL DIFFERENTIAL EQUATIONS

MARGUS LILLEMÄE, ARVET PEDAS, MIKK VIKERPUUR

Institute of Mathematics and Statistics, University of Tartu

Narva 18, Tartu, 51009, Estonia

E-mail: `margus.lillemae@ut.ee, arvet.pedas@ut.ee, mikk.vikerpuur@ut.ee`

We consider a fractional initial value problem

$$(\mathrm{D}^{\alpha_2}_{Cap}u)(t)+r_1(t)(\mathrm{D}^{\alpha_1}_{Cap}u)(t) + r_0(t)u(t) = f(t), \quad t \in [0,1], \tag{1}$$

$$u(0) = u_0, \quad u_0 \in \mathbb{R} := (-\infty, \infty), \tag{2}$$

where $0 < \alpha_1 < \alpha_2 < 1$, and the functions r_0, r_1 and f are continuous: $r_0, r_1, f \in C[0,1]$. Here $\mathrm{D}^{\alpha}_{Cap}u$ is an α-order Caputo fractional derivative of the unknown function $u = u(t)$. Then (see [1]) problem (1)–(2) possesses a unique solution $u \in C[0,1]$ such that $\mathrm{D}^{\alpha_2}_{Cap}u \in C[0,1]$. However, we cannot in general expect the solution u to belong to $C^q[0,1]$ (that is, u is q times continuously differentiable on interval $[0,1]$) for r_0, r_1 and $f \in C^q[0,1]$, $q \in \mathbb{N} := \{1, 2, \ldots\}$. Instead, we can show that if $r_0, r_1, f \in C^{q,\mu}(0,1]$, $q \in \mathbb{N}$, $\mu \in \mathbb{R}$, $\mu < 1$, then u and its derivative $\mathrm{D}^{\alpha_2}_{Cap}u$ belong to $C^{q,\nu}(0,1]$, where $\nu = \max\{1-(\alpha_2-\alpha_1), \mu\}$ [1]. Here, by $C^{q,\mu}(0,1]$ ($q \in \mathbb{N}$, $\mu \in \mathbb{R}$, $\mu < 1$) we denote the set of functions $u \in C[0,1] \cap C^q(0,1]$ such that

$$|u^{(i)}(t)| \le c \left\{ \begin{array}{ll} 1 & \text{if } i < 1-\mu \\ 1 + |\log t| & \text{if } i = 1-\mu \\ t^{1-\mu-i} & \text{if } i > 1-\mu \end{array} \right\}, \quad 0 < t \le 1, \quad i = 1, \ldots, m,$$

where c is a positive constant independent of t. Thus, when constructing high order numerical methods for problem (1)-(2), one should take into account, in some way, the possible non-smooth behaviour of the exact solution.

We propose a method for solving (1)-(2) based on improving the boundary behavior of the exact solution with the help of a change of variables, and on central part interpolation by polynomial splines on the uniform grid, which was introduced in [2] for the numerical solution of weakly singular Fredholm integral equations of the second kind. We adapt this approach for the problem (1)-(2) and derive global error estimates for the approximate solution [3].

REFERENCES

[1] A. Pedas, M. Vikerpuur. Spline collocation for multi-term fractional integro-differential equations with weakly singular kernels. *Fractal Fract.*, **5** :90, 2021.

[2] K. Orav-Puurand, G. Vainikko. Central part interpolation schemes for integral equations. *Numerical Functional Analysis and Optimization*, **30** :352–370, 2009.

[3] M. Lillemäe, A. Pedas, M. Vikerpuur. Central part interpolation schemes for a class of fractional initial value problems. *Acta Comment. Univ. Tartu. Math*, **26** :161–178, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

A GENERALIZED BOHR-JESSEN TYPE THEOREM FOR THE EPSTEIN ZETA-FUNCTION

RENATA MACAITIENĖ

Institute of Regional Development, Šiauliai Academy, Vilnius University

Vytauto street 84, LT-76352 Šiauliai, Lithuania

Faculty of Business and Technologies, Šiauliai State Higher Education Institution

Aušros av. 40, LT-76241 Šiauliai, Lithuania

E-mail: renata.macaitiene@sa.vu.lt

In the talk, some aspects on the value-distribution of the Epstein zeta-function will be discussed. Let Q be a positive definite quadratic $n \times n$ matrix and $Q[\underline{x}] = \underline{x}^{\mathrm{T}} Q \underline{x}$ for $\underline{x} \in \mathbb{Z}^n$. The Epstein zeta-function $\zeta(s;Q)$, $s = \sigma + it \in \mathbb{C}$, is defined, for $\sigma > \frac{n}{2}$, by the series

$$\zeta(s;Q) = \sum_{\underline{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} (Q[\underline{x}])^{-s},$$

and can be continued analytically to the whole complex plane, except for a simple pole at the point $s = \frac{n}{2}$ with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$. The probabilistic limit theorem of Bohr-Jessen type for the Epstein zeta-function $\zeta(s;Q)$ with even $n \geq 4$ and integers $Q[\underline{x}]$ was published in [1]. Namely, we have proved that, on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$, there exists an explicitly given probability measure $P_{Q,\sigma}$ such that, for $\sigma > \frac{n-1}{2}$,

$$\frac{1}{T} \mathrm{meas}\big\{t \in [0,T] : \zeta(\sigma + it; Q) \in A\big\}, \quad A \in \mathcal{B}(\mathbb{C}), \tag{1}$$

converges weakly to $P_{Q,\sigma}$ as $T \to \infty$. Here meas$A$ denotes the Lebesgue measure of a measurable set $A \subset \mathbb{R}$, and $\mathcal{B}(\mathbb{C})$ – the Borel σ-field of the space $\mathbb{C}$.

We will discuss more general case, i. e., in place of (1), the weak convergence for

$$\frac{1}{T} \mathrm{meas}\big\{t \in [0,T] : \zeta(\sigma + i\varphi(t); Q) \in A\big\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

where $\varphi(t)$ is a differentiable function with a monotonic derivative, will be analyzed. This result was obtained jointly with A. Laurinčikas in [2].

Acknowledgment. The research is funded by the Research Council of Lithuania, agreement No S-MIP-22-81.

REFERENCES

[1] A. Laurinčikas, R. Macaitienė. A Bohr-Jessen type theorem for the Epstein zeta-function. *Results in Math.*, **73** (4):147–163, 2018.

[2] A. Laurinčikas, R. Macaitienė. A generalized Bohr-Jessen type theorem for the Epstein zeta-function. *Mathematics*, **10** (12):1–11, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

SUPER-RESOLUTION DIFFUSION WEIGHTED IMAGING FROM HIGHLY COMPRESSIVELY SENSED MEASUREMENTS

KRZYSZTOF MALCZEWSKI

Institute of Information Technology, Warsaw University of Life Sciences

Nowoursynowska / building 34, 02-776 Warsaw, Poland

E-mail: `krzysztof_malczewski@sggw.edu.pl`

The purpose of this paper is to present a new, rapid compression-sensed magnetic resonance image reconstruction and enhancement technique for diffusion imaging. This study's primary objective is to eliminate two significant obstacles of diffusion MR image reconstruction techniques, i.e. combining a highly-sparse k-q space sampling pattern with super-resolution (SR) image enhancement circumvents the limitations of image resolution and algorithm reconstruction time. The algorithm utilizes the Wasserstein Generative Adversarial Networks (WGAN) framework. Moreover, the proposed method combines a highly compressively sensed k-space with deformable motion registration module to fuse adjacent frames in order to effectively use detailed information in multiple consecutive frames, and improves the spatio-temporalality of low-resolution images in sequential images. WGANs are initially taught to map zero-filling images onto full-sample images. Then, the zero-filled portion of the k-space data is retrieved [1]. The proposed method was introduced to optimize model training while GAN was used to improve the effect of image high-frequency texture detail reconstruction [2]. The $\Psi_G = \{W_{1:L}; b_{1:L}\}$ represents the weight and deviation of the L-layer-deep network and is obtained by optimizing the SR generation network's loss function l_G [3].

$$\Psi_G^* = \underset{\Psi_G}{argmin} \frac{1}{N} \sum_{t=1}^{N} l_G(G_{\Psi_G} I_t^{LR}, I_t^{HR}).$$

The loss function of the entire network, according to the SRGAN, includes the loss functions of the generator l_G and discriminator l_D:

$$l_D = \frac{1}{N} \sum_{n=1}^{N} \Big(log\big(1 - D_{\Psi_D}(G_{\Psi_G} I^{SR})\big)\Big) - log\Big(D_{\Psi_D}(I^{HR})\Big).$$

Here, N is the number of target images, and $D_{\Psi_D}(G_{\Psi_G} I^{SR}))$ and $D_{\Psi_D}(I^{HR})$ are the reconstructions for the discriminator's generator $G_{\Psi_G} I^{SR})$ and the original image I^{HR}, respectively.

REFERENCES

[1] K. Malczewski. Rapid diffusion weighted imaging with enhanced resolution. *Applied Magnetic Resonance*, **51** (3):221–239, 2020.

[2] Y. Chen, et al. Efficient and accurate MRI super-resolution using a generative adversarial network and 3D multi-level densely connected network. *ICMICCAI*, 91–99, 2019.

[3] Z. Chen, Y. Tong. Face super-resolution through wasserstein gans. *arXiv preprint*, arXiv: 1705.02438, 2017.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

MODELING THE EUROPEAN UNION EMISSION TRADING PRICES TIME-SERIES BY GENERATIVE ADVERSARIAL NETWORKS WITH INBUILT COMPRESSED SENSING PARADIGM BASED TRAINING SETS DENOISING

KRZYSZTOF MALCZEWSKI

Institute of Information Technology, Warsaw University of Life Sciences

Nowoursynowska / building 34, 02-776 Warsaw, Poland

E-mail: krzysztof_malczewski@sggw.edu.pl

Carbon prices follow a stochastic process of complex time series with nonstationary and nonlinear properties, making the forecasting of $CO2$ emission prices a crucial and difficult issue for policy-makers and market participants.The presented method combines the generative adversarial networks with the compressed sensing based test and training sets denoising paradigm. Autoencoders use unsupervised learning to extract features from high-dimensional data and reconstruct them from representations of those features. They consist of encoding and decoding procedures, with encoding mapping the input to the feature spaces and decoding returning the spaces to their original state. There are two NNs for the GAN structure: Generator (G) and Discriminator (D). The G and D engage in a competition wherein G develops candidates and D evaluates them. The purpose of G's training is to enhance D's error rate. The training could be described by the classical value function in the way below:

$$\min_G \max_D = E_{x \ p_{data}(x)} \left[\log \left(D\left(x\right) \right) \right] + E_{z \ p_z(z)} \left[\log \left(1 - D\left(G\left(z\right) \right) \right) \right].$$

The G is a differentiable function represented by a multilayer perceptron (MLP), and $D(x)$ is the probability that x originates from the data rather than p. Simultaneously, the maximization of assigning correct labels for D and the minimization of $log\left(1 - D\left(G\left(z\right)\right)\right)$ are trained.

The algorithm also utilizes the Compressed Sensing theorem [3]. Initial data filtering takes advantage of sparsity. This is based on the assumption that orthogonal data is redundant. The CSD is first carried out using the ANN-learning paradigm [1]. It has been shown that the GAN based forecasting scheme combined with modified learning paradigm is an excellent method for enhancing model prediction performance by reducing the level of noise detected in EU ETS price data.

REFERENCES

[1] K. Malczewski, Z. Krysiak, A. Markiewicz. Hybrid convolutional neural networks and modified GARCH model based the European Union's emissions trading scheme prices forecasting. In: *The proceedings of the World Congress in Computer Science, Computer Engineering, and Applied Computing*, CSCE, 2022.

[2] S. Bai, J.Z. Kolter, V. Koltun. An empirical evaluation of generic convolutional and recurrent networks for sequence modeling. *Applied Magnetic Resonance*, arXiv:1803.01271, 2018.

[3] J. Jin, B. Yang, K. Liang, X. Wang. General image denoising framework based on compressive sensing theory. *C and Graphics*, **38** :382–391, 2014.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

FUZZY LOGIC IN SOME MACHINE LEARNING METHODS

VALĒRIJS MIHAILOVS

University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: `valerijs.mihailovs@lu.lv`

In this work, the usage of fuzzy equivalence relations aggregation to determine the similarities of objects in the clustering process will be shown [1]. Different t-norms, such as Lukasiewicz t-norm, product t-norm, and Hamacher t-norm, are used. It is possible to assign different weights to attributes, thus defining the importance of attributes in the decision-making process. Also, we will demonstrate how membership functions can be used in classification problems to calculate the degree of membership of each data point in each class. This allows for making classification decisions based on these membership values. Such an approach enables a more nuanced and flexible classification process that can take into account the inherent uncertainty and variability present in many real-world datasets. Membership functions can be employed in various classification algorithms, such as fuzzy k-nearest neighbors, fuzzy decision trees, and fuzzy neural networks. These methods can provide improved performance in situations where traditional classification techniques may struggle due to data uncertainty or imprecision. Furthermore, the work presents the performance of these methods on different datasets.

REFERENCES

[1] O. Grigorenko, V. Mihailovs. Aggregated fuzzy equivalence relations in clustering process. *CCIS*, **1601** :448-459, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

JOINT APPROXIMATION BY LERCH ZETA-FUNCTIONS

TOMA MIKALAUSKAITĖ[1], DARIUS ŠIAUČIŪNAS[2]

[1]*Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: `toma.mikalauskaite@mif.stud.vu.lt`

[2]*Šiauliai Academy, Vilnius University*

P. Višinskio street 25, LT-76351 Šiauliai, Lithuania

E-mail: `darius.siauciunas@sa.vu.lt`

Let $s = \sigma + it$ be a complex variable, and $0 < \alpha \leqslant 1$ and λ be real parameters. The Lerch zeta-function $L(\lambda, \alpha, s)$ is defined, for $\sigma > 1$, by the series $L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \mathrm{e}^{2\pi i \lambda m}(m+\alpha)^{-s}$, and has the meromorphic continuation to the whole complex plane. The function $L(\lambda, \alpha, s)$, as other zeta-functions, for some classes of the parameters λ and α, is universal in the sense that its shifts $L(\lambda, \alpha, s + i\tau)$, $\tau \in \mathbb{R}$, approximate every analytic function defined on the strip $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$. Some results on approximation of analytic functions by shifts $L(\lambda, \alpha, s + i\tau)$ for all parameters λ and α were obtained in [1] and [2].

In the report, we generalize the above results for a collection of Lerch zeta-functions $L(\lambda_1, \alpha_1, s), \dots, L(\lambda_r, \alpha_r, s)$ with arbitrary parameters $\lambda_1, \dots, \lambda_r$ and $\alpha_1, \dots, \alpha_r$. Let $\underline{\lambda} = (\lambda_1, \dots, \lambda_r)$, $\underline{\alpha} = (\alpha_1, \dots, \alpha_r)$, and $H(D)$ be the space of analytic on D functions equipped with the topology of uniform convergence on compacta.

THEOREM 1. *Suppose that* $0 < \lambda_j \leqslant 1$ *and* $0 < \alpha_j \leqslant 1$ *are arbitrary,* $j = 1, \dots, r$. *Then there exists a non-empty closed set* $F_{\underline{\lambda},\underline{\alpha}} \subset H^r(D)$ *such that, for compact sets* $K_1, \dots, K_r \subset D$, $(f_1(s), \dots, f_r(s)) \in F_{\underline{\lambda},\underline{\alpha}}$, *and every* $\varepsilon > 0$,

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas}\left\{\tau \in [0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |L(\lambda_j, \alpha_j, s + i\tau) - f_j(s)| < \varepsilon\right\} > 0.$$

Theorem 1 has a discrete version. Let $\underline{h} = (h_1, \dots, h_r)$, $h_j > 0$, $j = 1, \dots, r$.

THEOREM 2. *Under hypotheses of Theorem 1, there exists a non-empty closed set* $F_{\underline{\lambda},\underline{\alpha},\underline{h}} \subset H^r(D)$ *such that, for compact sets* $K_1, \dots, K_r \subset D$, $(f_1(s), \dots, f_r(s)) \in F_{\underline{\lambda},\underline{\alpha},\underline{h}}$, *and every* $\varepsilon > 0$,

$$\liminf_{N\to\infty} \frac{1}{N+1} \# \left\{0 \leqslant k \leqslant N : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |L(\lambda_j, \alpha_j, s + ikh_j) - f_j(s)| < \varepsilon\right\} > 0.$$

REFERENCES

[1] A. Laurinčikas. "Almost" universality of the Lerch zeta-function. *Math. Commun.*, **24** (1):107–118, 2019.

[2] A. Rimkevičienė, D. Šiaučiūnas. On discrete approximation of analytic functions by shifts of the Lerch zeta-function. *Mathematics*, **10** (24):art. no. 4650, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

VISCOELASTIC MODELS OF TISSUES; MATHEMATICAL BACKGROUND AND HYSTERESIS LOOPS

MÁRIA MINÁROVÁ, SOŇA ZAJÍCOVÁ, KATARÍNA TVRDÁ

Slovak University of Technology

Radlinkského 11, Bratislava 81005, Slovakia

E-mail: `maria.minarova@stuba.sk, katarina.tvrda@stuba.sk, zajicova.sona.sz@gmail.com`

Human, animal and plant tissues are typically viscoelastic. Their response to a mechanical load is neither purely viscous, nor purely elastic, but somewhere between. We say they exhibit both viscous and elastic properties. The models standing behind such materials are called viscoelastic models. Their mechanical behaviour is governed by stress σ and strain ε relation, called physical or constitutive equation, as well. Physical relations of (H) - Hookean elastic and (N) Newtonian viscous members are respectively: (H): $\sigma = E\varepsilon$ and (N): $\sigma = \eta\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}$. In viscoelastic models, stress and strain are both functions of time. Parallel "|" and/or serial "-" connection is exploited when creating various viscoelastic models. Arisen configurations then induce corresponding geometric equations. Geometric equations are of a great importance herein: In *parallel connection* of (H) and (N), both members move equally, $\varepsilon = \varepsilon_H = \varepsilon_N$; summing up of stress functions of involved members yields the total stress function of the model $\sigma = \sigma_H + \sigma_N$. On the other hand, from *serial connection* we have geometric equations $\varepsilon = \varepsilon_H + \varepsilon_N$ and $\sigma = \sigma_H = \sigma_N$. Stress-strain relation of entire model can be then derived from corresponding geometric and physical equations by eliminating all variables indexed by letters H and N. In viscoelastic case, it is always a differential equation. Moreover, the difference between the mechanical response during loading and unloading is called hysteresis, [1]. Tissues can be represented by less or more complex viscoelastic models. As an example, human lung parenchyma can be taken – with the model standing behind it, of its structural form, [2]

$$(H_2)|[(H_1) - (N_1)]$$

and constitutive relation

$$\eta_1(E_1 + E_2)\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} + E_1E_2\varepsilon(t) = \eta_1\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} + E_2\sigma(t).$$

Having the viscoelastic model at hand, the theoretical investigation can be done, validated with lab tests and used for behaviour prediction of materials' response to various load.

Acknowledgment. This research is supported by grants VEGA 1/0036/23, VEGA 1/0453/20, VEGA 1/0155/23 and APVV-18-0052.

REFERENCES

[1] P. Krejčí. Hysteresis, convexity and dissipation in hyperbolic equations. *Gakuto Intern. Ser. Math. Sci. Appl.*, **8** Tokyo, 1996.

[2] F. G. Hoppin, J. Hildebrandt. *Mechanical Properties of the Lung. Bioengineering Aspects of the Lung.* University of California, 1998.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

COMPARATIVE ANALYSIS OF MODELS OF GENETIC AND NEURONAL NETWORKS

DIANA OGORELOVA[1], FELIX SADYRBAEV[2]

[1]*Daugavpils University*
Parades street 1, LV-5401, Daugavpils, Latvia

[2]*Institute of Mathematics and Computer Science, University of Latvia*
Raina blvd. 29, Riga LV-1459, Latvia
E-mail: `diana.ogorelova@du.lv, felix@latnet.lv`

Genetic networks can me modeled by the system of ordinary differential equations

$$\begin{cases} x_1' = f_1(w_{11}x_1, ..., w_{1n}x_n, \theta_1) - v_1 x_1, \\ ..., \\ x_n' = f_n(w_{n1}x_1, ..., w_{nn}x_n, \theta_n) - v_n x_n, \end{cases}$$

where $f_i(w_{i1}x_1, ..., w_{in}x_n) = e^{-e^{-\mu_i(w_{i1}x_1+\dots+w_{in}x_n-\theta_i)}}$ are Gompertz functions.

In the theory of neuronal networks the system of ODE

$$\begin{cases} x_1' = \tanh(a_{11}x_1 + ... + a_{1n}x_n) - b_1 x_1, \\ ..., \\ x_n' = \tanh(a_{n1}x_1 + ... + a_{nn}x_n) - b_n x_n, \end{cases}$$

is used to model a network.

We make comparative analysis of both systems for particular cases of dimensions 2, 3 and 4. In a focus of the research are attractors of both systems. We construct examples of attractors of different nature, including stable equilibria and limit cycles.

REFERENCES

[1] D. Ogorelova, F. Sadyrbaev. On a three-dimensional neural network model. *Vibroengineering PROCEDIA*, **47** :69–73, 2022.

[2] I. Samuilik, F. Sadyrbaev, D. Ogorelova. Mathematical modeling of three-dimensional genetic regulatory networks using logistic and Gompertz functions. *WSEAS Transactions on Systems and Control*, **17** :101–107, 2022.

[3] D. Ogorelova, F. Sadyrbaev, V. Sengileyev. Control in inhibitory genetic regulatory network models. *WSEAS Transactions on Systems and Control*, **1** (5): 393–400, 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

AGGREGATION OF CREDIT RISK LEVELS BASED ON FUZZY CLUSTERING

PAVELS ORLOVS[1], SVETLANA ASMUSS[2,3]

[1] *BluOr Bank AS*

Smilsu street 6, Riga LV-1050, Latvia

E-mail: `pavels.orlovs@gmail.com`

[2] *Department of Mathematics, University of Latvia*

Jelgavas street 3, Riga LV-1004, Latvia

[3] *Institute of Mathematics and Computer Science, University of Latvia*

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `svetlana.asmuss@lu.lv`

Our research deals with a special construction of aggregation, which is based on fuzzy clustering. The need for aggregation operators based on similarities appears, for example, in decision making if values to be aggregated represent evaluations provided by several experts. In this talk we develop and apply in risk management the approach proposed in our previous works [1], [2] where such similarities were described by fuzzy equivalence relations.

Suppose we have an initial data set (learning set) of customers with different attributes that somehow characterize these customers. Suppose the experts have assessed the levels of credit risk for these customers using their own analytical approaches and skills. We propose a special design aimed at involving the results of fuzzy clustering in the aggregation of risk levels given by the experts. For this aim, fuzzy clustering of the learning set based on customer attributes is used. The inclusion of risk level assessments given by the experts for all customers of a cluster in the process of aggregation of risk level assessments given by the experts for one object of this cluster allows to obtain a more objective and unbiased final aggregated credit rating.

REFERENCES

[1] P. Orlovs, S. Asmuss. General aggregation operators based on a fuzzy equivalence relation in the context of approximate systems. *Fuzzy Sets Syst.*, **291** :114–131, 2016.

[2] S. Asmuss, P. Orlovs. Fuzzy metric approach to aggregation of risk levels. *Studies in Computational Intelligence*, **819** :175–181, 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

VALIDATION OF THE TWO-SAMPLE LOCATION-SCALE MODEL

LEONORA PAHIRKO, JANIS VALEINIS

Faculty of Physics, Mathematics and Optometry, University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: `leonora.pahirko@lu.lv, janis.valeinis@lu.lv`

The two sample problems probably are the most common in statistics. In practice for comparison of two groups usually some measure of central tendency is compared, for example, using t-test or Wilcoxon test. However, the comparison of the whole distribution would be preferable and could lead to more subtle conclusions on the observed differences between two groups compared to the above mentioned two-sample tests.

In medicine to declare a positive treatment effect for all population members we could test for the location model $F_1(t) = F_2(t - \mu)$, $t \in \mathbb{R}$, between two distribution functions F_1 and F_2. It indicates a uniform shift between two samples by some constant μ. Or else, if the equality of two means or medians is not rejected, there could still be the differences between the distributions of both samples regarding their scales, indicating non-equal treatment effect for all members of the population. The scale model between two distribution functions is expressed as $F_1(t) = F_2(t/\sigma)$, $t \in \mathbb{R}$, where σ is a scale parameter. Taken both mentioned semi-parametric models together, the two-sample location-scale model is defined as

$$F_1(t) = F_2\left(\frac{t-\mu}{\sigma}\right), \quad t \in \mathbb{R}.$$

Numerous non-parametric tests for simultaneous test of the location and scale differences under the the two-sample location-scale model can be found in the literature (see for example [1]). In order to apply those tests the two-sample location-scale model needs to be validated beforehand. Doksum and Sievers [2] introduced simultaneous confidence bands for the general shift function, which is a graphical tool for validation of the location and the location-scale models. Hall et.al. [3] proposed a test based on the empirical characteristic functions to verify if the two samples belong to the same location-scale model family.

We propose to use the empirical likelihood-based test for the probability-probability plot to validate the location-scale model between two samples. This can be done by testing the equality of both distributions using the location and scale parameter estimation to transform one of the samples under the location-scale assumption.

REFERENCES

[1] M. Marozzi. Nonparametric simultaneous tests for location and scale testing: a comparison of several methods. *Communications in Statistics-Simulation and Computation*, **42** (6):1298–1317, 2013.

[2] K.A. Doksum, G.L. Sievers. Plotting with confidence: Graphical comparisons of two populations. *Biometrika*, **63** (3):421–434, 1976.

[3] P. Hall, F. Lombard, C.J. Potgieter. A new approach to function-based hypothesis testing in location-scale families. *Technometrics*, **55** (2):215–223, 2013.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ASYMPTOTIC ANALYSIS AND NUMERICAL SOLUTIONS OF STATIONARY FLOWS IN NETWORK OF VESSELS

GRIGORY PANASENKO

UMR CNRS 5208, University Jean Monnet, Saint-Etienne, France and Institute of Applied Mathematics, Vilnius University, Lithuania

23 rue P.Michelon, 42023, Saint-Etienne, France; Naugarduko street 24, Vilnius, Lithuania

E-mail: `grigory.panasenko@univ-st-etienne.fr`

The talk briefly presents the results of asymptotic analysis for non-Newtonian flows in thin tube structures [1], [2]. The computation of the leading term approximation of the solution is related to the equation on the graph, which is an elliptic nonlinear problem. We introduce a numerical method to solve the equation on the graph and apply it to the realistic network of vessels. The results are obtained in collaboration with Kristina Kaulakyte, Nikolajus Kozulinas, Konstantinas Pileckas, Vytenis Sumskas.

REFERENCES

[1] G. Panasenko, K. Pileckas, B. Vernescu. Steady state non-Newtonian flow with strain rate dependent viscosity in thin tube structure with no slip boundary condition. *Mathematical Modelling of Natural Phenomena*, **17** (18), 2022. www.mmnp-journal.org (open access)

[2] G. Panasenko, K. Pileckas. Partial asymptotic reduction for the steady-state non-Newtonian flow with strain rate dependent viscosity in thin tube structure. *J. Math. Fluids Mechanics*, **25** (11), 2023.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

NUMERICAL SOLUTION OF LINEAR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

ARVET PEDAS, MIKK VIKERPUUR

Institute of Mathematics and Statistics, University of Tartu

Narva 18, Tartu, 51009, Estonia

E-mail: `arvet.pedas@ut.ee, mikk.vikerpuur@ut.ee`

We consider a class of fractional weakly singular integro-differential equations

$$
\begin{aligned}
(D_{Cap}^{\alpha_p} y)(t) &+ \sum_{i=0}^{p-1} d_i(t)(D_{Cap}^{\alpha_i} y)(t) \\
&+ \sum_{i=0}^{q-1} \int_0^t (t-s)^{-\kappa_i} K_i(t,s)(D_{Cap}^{\theta_i} y)(s)ds = f(t), \quad 0 \le t \le b,
\end{aligned}
\tag{1}
$$

subject to the initial conditions

$$
y^{(i)}(0) = \gamma_i\,, \quad i = 0, \dots, n-1. \tag{2}
$$

Here D_{Cap}^{δ} is the Caputo differential operator of order $\delta > 0$ and $n := \lceil \alpha_p \rceil$ is the smallest integer greater or equal to the highest fractional order α_p. We assume that $0 \le \alpha_0 < \alpha_1 < \cdots < \alpha_p \le n$, $0 \le \theta_j < \alpha_p$, $0 \le \kappa_j < 1$ $(j = 0, \dots, q-1)$ with $p,\, q \in \{1, 2, \dots\}$, and that the given functions d_i $(i = 0, \dots, p-1)$, K_j $(j = 0, \dots, q-1)$ and f are continuous on their respective domains.

Following [1], we reformulate (1)–(2) as a Volterra integral equation of the second kind with respect to the fractional derivative $D_{Cap}^{\alpha_p} y$. We then regularize the solution by a suitable smoothing transformation and solve the transformed integral equation by a piecewise polynomial collocation method on a mildly graded or uniform grid. We show the convergence of the proposed algorithm and present global superconvergence results for a class of specific collocation parameters. Finally, we complement the theoretical results with some numerical examples.

REFERENCES

[1] A. Pedas, M. Vikerpuur. Spline collocation for multi-term fractional integro-differential equations with weakly singular kernels. *Fractal and Fractional*, **5(3)** 90, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

HOMOGENIZATION OF ALTERNATING NONLINEAR BOUNDARY CONDITIONS, WITH HIGH CONTRASTS

MARIA-EUGENIA PÉREZ-MARTÍNEZ

Universidad de Cantabria

Av. de los Castros s/n 39006 Santander, Spain

E-mail: `meperez@unican.es`

We address a homogenization problem for the Laplace operator posed in a bounded domain of the upper halfspace, a part of its boundary being in contact with the plane. On this part, the boundary conditions alternate from Neumann to nonlinear-Robin, being of Dirichlet type outside. The nonlinear-Robin boundary conditions are imposed on small regions periodically placed along the plane and contain a Robin parameter that can be very large. Depending on the different relations between parameters (period, size of the regions and Robin parameter), a nonlinear capacity term may arise in the strange term which depends on the macroscopic variable and allows us to extend the usual capacity definition to semilinear boundary conditions, cf. [2]. Extensions to strainer Winkler foundations are considered, focusing mainly on the averaged Robin-Winkler boundary condition, cf. [1] and [3].

REFERENCES

[1] D. Gómez, S.A. Nazarov, M.-E. Pérez-Martínez. Asymptotics for spectral problems with rapidly alternating boundary conditions on a strainer Winkler foundation. *Journal of Elasticity*, **142** :89–120, 2020.

[2] D. Gómez, M.-E. Pérez-Martínez. Boundary homogenization with large reaction terms on a strainer-type wall. *Z. Angew. Math. Phys.*, **73** :234, 2022. https://doi.org/10.1007/s00033-022-01869-8

[3] D. Gómez, M.-E. Pérez-Martínez. Spectral homogenization problems in linear elasticity: the averaged Robin reaction matrix. In: *Proc. of The 16th International Conference on Integral Methods in Science and Engineering*, to appear, 2023.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

METHOD OF F-TRANSFORMS TO THE VOLTERRA INTEGRAL EQUATION WITH A WEAKLY SINGULAR KERNEL

IRINA PERFILIEVA, THI MINH TAM PHAM

University of Ostrava, Institute for Research and Applications of Fuzzy Modeling

30. dubna 22, 701 03 Ostrava 1, Czech Republic

E-mail: Irina.Perfilieva@osu.cz

The linear Volterra equation whose general form is

$$u(t) = g(t) + \lambda \int_0^t k(t,s)u(s)ds, \qquad t \in [0,1], \tag{1}$$

belongs to the group of classical fundamental equations. In (1), $\lambda \in \mathbb{R}\setminus\{0\}$, *kernel* $k : [0,1]\times[0,1] \to \mathbb{R}$, and *source* $g : [0,1] \to \mathbb{R}$ are given, and $u : [0,1] \to \mathbb{R}$ is unknown. If k is integrable and bounded and $g \in L^2[0,1]$, then (1) always has a unique solution.

We will focus on *weak singular kernels*, which are represented by $K(t,s) = \alpha(t,s)|t-s|^{-\nu}$, where $0 < \nu < 1$, and $\alpha : [0,1]\times[0,1] \to \mathbb{R}$ is smooth above the diagonal. Generalizing, we say that a kernel K is weakly singular [1], if it is absolutely integrable with respect to s and satisfies $\sup_{[0,1]} \int_0^1 |k(t,s)|ds < \infty$.

In the case of (1) with a weakly singular kernel the main problem in the development of numerical methods is the discontinuity of the kernel on the diagonal of the integration domain. In this contribution, we show that in this particular case the (fuzzy) F-transform method [2] is applicable and occupies a special place among other combined methods.

In more detail, to find a numerical solution of (1), all involved functional objects are replaced by their approximations in the form of inverse F-transforms. After this step, integration in (1) becomes applicable only to the basic functions of a uniform fuzzy partition, which does not depend on any particular type of integrand. As a result, the action of integration in (1) is focused on what can be associated with a certain spatial structure (fuzzy partition), and separated from the functional objects in (1).

We have calculated the operational matrix of the Volterra operator by applying the integral operator to the basic functions of a fuzzy partition, and reduced (1) to a system of linear equations with a (non-degenerate) triangular matrix of coefficients. An easily found solution to this system gives the F-transform components of the unknown function and its approximation in the form of the inverse F-transform.

We proved the convergence theorem and gave estimations of the numerical complexity of the proposed method.

REFERENCES

[1] G. Vainikko. *Multidimensional Weakly Singular Integral Equations.* Springer, Berlin, 1993.

[2] I. Perfilieva. Fuzzy transform: theory and applications. *Fuzzy Sets and Systems*, **157** :9930–1023, 2006.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON THE CONVERGENCE OF THE DIFFERENCE SCHEME FOR NONLINEAR ELLIPTIC EQUATION WITH INTEGRAL BOUNDARY CONDITION

KRISTINA PUPALAIGĖ[1], REGIMANTAS ČIUPAILA[2], GAILĖ ŠALTENIENĖ[3], MIFODIJUS SAPAGOVAS[4]

[1] *Department of Applied Mathematics, Kaunas University of Technology*
Studentų street 50, LT-51368, Kaunas, Lithuania
E-mail: `kristina.pupalaige@ktu.lt`
[2] *Vilnius Gediminas Technical University*
Saulėtekio av. 11, LT-10223, Vilnius, Lithuania
E-mail: `regimantas.ciupaila@vilniustech.lt`
[3] *Vilnius Gediminas Technical University*
Saulėtekio av. 11, LT-10223, Vilnius, Lithuania
E-mail: `gaile.kamile.salteniene@vilniustech.lt`
[4] *Institute of Data Science and Digital Technologies, Vilnius University*
Akademijos street 4, LT-08412, Vilnius, Lithuania
E-mail: `mifodijus.sapagovas@mif.vu.lt`

We investigate the boundary value problem

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y,u), \quad (x,y) \in \Omega = \{0 < x < 1, 0 < y < 1\},\\
&u(x,0) = \mu_1(x), \quad u(x,1) = \mu_2(x), \quad u(0,y) = \mu_3(y),\\
&u(1,y) = \gamma \int\limits_{\xi}^{1} u(x,y)dx + \mu_4(y).
\end{aligned}
$$

It was analyzed with which values of parameters ξ and γ the matrix of the system of difference equations is an M–matrix. Applying the M–matrix theory, the convergence conditions of iterative methods for solving the system of nonlinear difference equations were found. The main result of the investigation is that the majorant function is constructed and the convergence of the finite difference method is proved.

REFERENCES

[1] K. Bingelė, A. Bankauskienė, A. Štikonas. Spectrum curves for a discrete Sturm-Lioville problem with one integral boundary conditionl. *Nonlin. Anal. Model. Control.*, **24** :755–744, 2019.

[2] M. Sapagovas, O. Štikonienė, K. Jakubėlienė, R. Čiupaila. Finite difference method for boundary value problem for nonlinear elliptic equation with nonlocal conditions. *Bound. Value Probl.*, **2019** (94):1–16, 2019.

[3] K. Pupalaigė M. Sapagovas, R. Čiupaila. Nonlinear elliptic equation with nonlocal integral boundary condition depending on two parameters. *Math. Model. Anal.*, **27** (4):610–628, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

MODELING OF PHOTONIC CRYSTAL SURFACE-EMITTING LASERS

MINDAUGAS RADZIUNAS

Weierstrass Institute

Mohrenstrasse 39, 10117 Berlin, Germany

E-mail: Mindaugas.Radziunas@wias-berlin.de

Semiconductor diode lasers are small, efficient, and relatively cheap devices used in many modern applications. Multiple applications require emission powers exceeding several ten Watts from a single diode and up to a few kiloWatts from a combined laser system. In this presentation, we consider novel Photonic Crystal Surface-Emitting Lasers (PCSELs), which, in contrast to conventional high-power edge-emitting broad-area lasers (BALs), are capable of emitting high power (up to 80 W at the moment [1]) beams of nearly perfect quality in the (z) direction, perpendicular to the (x/y) plain of active material. The critical part of PCSELs, enabling an efficient coupling of within the active layer generated optical fields and their redirection along the z axis, is a properly constructed 2-dimensional photonic crystal layer. The model to be considered and integrated numerically [2]) is derived from Maxwell equations and is a 1 (time)+2 (space) dimensional system of PDEs for complex optical fields $u(t,x,y) = \binom{u^+}{u^-}$, $v(t,x,y) = \binom{v^+}{v^-}$, and real carrier density $N(t,x,y)$:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix} u \\ v \end{pmatrix} = \mathcal{H}(\beta(N))\begin{pmatrix} u \\ v \end{pmatrix} + F_{sp}, \quad \frac{\partial}{\partial t}N = D\nabla^2_{x,y}N + \mathcal{N}(N,|u|^2,|v|^2), \quad (x,y)\in[0,L]\times[0,L],$$

$$\text{where} \quad \mathcal{H}(\beta(N)) = -\begin{pmatrix} \sigma\frac{\partial}{\partial x} & \mathbf{0} \\ \mathbf{0} & \sigma\frac{\partial}{\partial y} \end{pmatrix} + \mathcal{I}\beta(N) + \mathbf{C}, \quad \sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{cases} u^+|_{x=0} = u^-|_{x=L} = 0, \\ v^+|_{y=0} = v^-|_{y=L} = 0. \end{cases}$$

Besides, an efficient location of several leading modes (Λ, Θ) of the related spectral problem [3],

$$\mathcal{H}(\bar\beta)\Theta(x,y) = \Lambda\Theta(x,y), \quad \Theta = \begin{pmatrix} \Theta_u \\ \Theta_v \end{pmatrix}, \qquad (\Theta_u, \Theta_v) \quad \text{satisfies boundary conditions,}$$

given by a system of four 2-D PDEs, is of great importance when designing PCSEL devices. We shall address the biggest challenges arising when treating above stated problems. Namely, a nontrivial construction of the implicitly defined 4×4 field coupling matrix $\mathbf{C}$, requiring a solution of the Helmholtz problem and multiple integrations of the calculated mode profile with different separately constructed exponentially growing and decaying Green's functions, as well as dealing with large discrete problems relating up to several million variables in large-emission-area (large L) PCSELs.

Acknowledgment. This work was carried out in the frame of the German Leibniz Association funded project "PCSELence" (funding number K487/2022).

REFERENCES

[1] T. Inoue et al. Self-evolving photonic crystals for ultrafast photonics. *Nat Commun*, **14** :50, 2023.

[2] T. Inoue et al. Comprehensive analysis of photonic-crystal surface-emitting lasers via time-dependent three-dimensional coupled-wave theory. *Phys. Rev. B*, **99** :035308, 2019.

[3] Y. Liang et al. Three-dimensional coupled-wave analysis for square-lattice photonic crystal surface emitting lasers with transverse-electric polarization: finite-size effects. *Optics Express*, **20** (14):15945, 2012.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

THE WADA INDEX, COEXISTING ATTRACTORS, AND SECRET IMAGES

MINVYDAS RAGULSKIS

Kaunas University of Technology

Studentu 50-146, Kaunas LT-51368, Lithuania

E-mail: `minvydas.ragulskis@ktu.lt`

The lakes of Wada are three (or more) disjoint connected open sets of the plane that all have the same boundary [1]. The visualization of Wada basins for several coexisting attractors in a form of a colormap where every point in the phase space of initial conditions is assigned to a different color became an important topic in computational analysis of nonlinear systems [2]. The uncertainty of a response to a perturbation of a nonlinear system possessing the Wada property (which is higher if compared to a fractal basin boundary) can be quantified by means of the Wada index [3]:

$$\omega_q^{(s)}(p_1, ..., p_m) = \frac{m}{\log(q)} I_q^{(s)} e^{(s)},$$

where s is the size of the observation window; q is the current color; m is the number of different colors in the current observation window; p_k is the discrete probability of the k-th color in the observation window, $k = 1, 2, ..., m$; $I_q^{(s)}$ is the indicator function of color q in the observation window; $e^{(s)}$ is the Shannon entropy of the current observation window.

The introduction of the concept of the perfect covering of the stego image helps to design the image sharing scheme based on the Wada index (the Wada index is directly exploited in the decoding scheme) [4]. A covering is a perfect covering if a sequential modification at the current observation window does not cause conflicts in all previous observation windows. It can be shown that a perfect covering does not exist for a carrier image with periodic boundary conditions. Also, it appears that the encoded stego image does not depend on the type of a perfect covering [4].

The proposed image hiding scheme based on the Wada index ensures that the secret image is not leaked from the modified bit planes with lowest indexes, the stego image is robust against RS steganalysis algorithms, and the payload capacity of the carrier image is comparable to best LSB schemes [4].

REFERENCES

[1] K. Yoneyama. Theory of continuous set of points (not finished). *Tohoku Math. J., First Series*, **12** :43–158, 1917.

[2] P. Ziaukas, M. Ragulskis. Fractal dimension and Wada measure revisited: no straightforward relationships in NDDS. *Nonlinear Dynamics*, **88** :871-882, 2017.

[3] L. Saunoriene, M. Ragulskis, J. Cao, M.A.F. Sanjuan. Wada index based on the weighted and truncated Shannon entropy. *Nonlinear Dynamics*, **104** :739-751, 2021.

[4] L. Saunoriene, M. Ragulskis. A steganographic scheme based on the Wada index. *Multimedia Tools and Applications*, https://doi.org/10.1007/s11042-023-14888-y, 2023.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

HEURISTIC CHOICE OF THE REGULARIZATION PARAMETER IN TIKHONOV METHOD

TOOMAS RAUS

Institute of Mathematics and Statistics, University of Tartu

Narva mnt 18, Tartu,51009, Estonia

E-mail: toomas.raus@ut.ee

We consider an operator equation

$$Au = f_*, \quad f_* \in R(A),$$

where $A \in L(H, F)$ is the linear continuous operator between real Hilbert spaces H and F. We assume that instead of the exact right-hand side f_* we have only an approximation $f \in F$. To get the regularized solution we consider Tikhonov method $u_\alpha = (\alpha I + A^*A)^{-1}A^*f$, where $\alpha > 0$ is the regularization parameter.

Well-known heuristic rule for choosing regularization parameter is the quasioptimality principle, where the parameter is chosen as the global minimum point of the function

$$\psi_Q(\alpha) = \alpha \left\| du_\alpha/d\alpha \right\| = \alpha^{-1} \left\| A^*(Au_{2,\alpha} - f) \right\|, \quad u_{2,\alpha} = (\alpha I + A^*A)^{-1}(\alpha u_\alpha + A^*f),$$

on the set of parameters $\Omega = \left\{ \alpha_j : \alpha_j = q\alpha_{j-1}, j = 1, 2, ..., M, 0 < q < 1 \right\}$. Unfortunately, this rule is unstable in this sense that it often fails in case of heat-type problems.

To get stable parameter choice rule we introduce modified quasioptimality criterion function $\psi_{\mathrm{MQ}}(\alpha)$ in the form:

$$\psi_{\mathrm{MQ}}(\alpha_0) = \psi_{\mathrm{Q}}(\alpha_0),$$

$$\psi_{\mathrm{MQ}}(\alpha_j) = \max\{\psi_{\mathrm{Q}}(\alpha_j), (d_{MD}(\alpha_j)/d_{MD}(\alpha_{j-1}))^{2/3}\psi_{\mathrm{MQ}}(\alpha_{j-1})\}, j > 0,$$

where the function $d_{MD}(\alpha) = \left\| B_\alpha(Au_\alpha - f) \right\|, \quad B_\alpha = \alpha^{1/2}(\alpha I + AA^*)^{-1/2}$. Choosing global minimum point of the function $\psi_{\mathrm{MQ}}(\alpha)$ as regularization parameter gives us stable heuristic rule, but to get more accuracy we present the following algorithm.

In article [1] is shown that at least one local minimum point m_k of the quasioptimality criterion function is always a good regularization parameter. Let α_{MQ} be the global minimum point of the function $\psi_{\mathrm{MQ}}(\alpha)$ on the set of local minimum points of the function $\psi_Q(\alpha)$.

Heuristic rule. For the regularization parameter choose the parameter

$$\alpha_H = \max\{\alpha_Q, \min\{\alpha_{MQ}, \alpha_{HR}\}\},$$

where α_Q and α_{HR} are the global minimum points of the functions $\psi_Q(\alpha)$ and $\psi_{HR}(\alpha) = \alpha^{-1/2}d_{MD}(\alpha)$, respectively.

REFERENCES

[1] T. Raus, U. Hämarik. Q-curve and area rules for choosing heuristic parameter in Tikhonov regularization. *Mathematics*, **8** (7, 1166):1–21, 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

DYNAMIC EQUATIONS ON TIME SCALES

ANDREJS REINFELDS

Institute of Mathematics and Computer Science, University of Latvia

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `reinf@latnet.lv`

In 1988 (Ph.D. thesis), Stefan Hilger [1] introduced the calculus of time scale in order to unify continuous and discrete analysis, see [2].

The equivalence (conjugacy) and the linearization problem in the theory of differential and difference equations (homeomorphisms, noninvertible mappings) was explored by many mathematicians.

In the present talk we generalize the reduction principle to the case of the dynamic systems on time scale in Banach space whose linear part split into two parts and satisfy weaker condition than exponential dichotomy.

We consider the dynamic system in a Banach space on unbounded above and unbounded below time scales

$$\left\{\begin{array}{rcl} x^{\Delta} & = & A(t)x + f(t,x,y), \\ y^{\Delta} & = & B(t)y + g(t,x,y). \end{array}\right. \tag{1}$$

This system satisfies the conditions of integral separation with the separation constant ν, nonlinear terms are unbounded and ε-Lipshitz, and the system has a trivial solution. Based on the integral version of the idea developed in the article [3] we generalize and find sufficient condition under which the system (1) is dynamic equivalent to

$$\left\{\begin{array}{rcl} x^{\Delta} & = & A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} & = & B(t)y + g(t,v(t,y),y). \end{array}\right. \tag{2}$$

In the case of unbounded above time scales we also find sufficient condition under which the noninvertible system is dynamic equivalent to

$$\left\{\begin{array}{rcl} x^{\Delta} & = & A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} & = & B(t)y + g(t,k(t,x,y),y). \end{array}\right. \tag{3}$$

REFERENCES

[1] S. Hilger. Generalized theorem of Hartman-Grobman on measure chains. *J. Austral. Math. Soc. Ser A*, **60** (2): 157–191, 1998.

[2] M. Bohner, A. Peterson. *Dynamic Equations on Time Scales. An Introduction with Applications.* Birkhäuser, Boston, Basel, Berlin, 2001.

[3] A. Reinfelds. The reduction principle for discrete and semidynamical systems in metric spaces. *Z. Angew. Mat. Phys.*, **45** (6): 933–955, 1994.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

VOLTERRA INTEGRODIFFERENTIAL EQUATIONS ON TIME SCALES

ANDREJS REINFELDS[1], SHRADDHA RAMANBHAI CHRISTIAN[2]

[1] *Institute of Mathematics and Computer Science, University of Latvia*
Raina blvd. 29, Riga LV-1459, Latvia
[2] *Department of Engineering Mathematics, Riga Technical University*
Zunda embankment 10, Riga LV-1048, Latvia
E-mail: `reinf@latnet.lv, Shraddha-Ramanbhai.Christian@rtu.lv`

Consider Volterra integrodifferential equation on an arbitrary unbounded time scale $\mathbb{T}$

$$x^{\Delta}(t) = f(t) + \int_a^t K(t, s, x(s), x^{\Delta}(s))\Delta s, \quad x(a) = x_0.$$

We define a new Volterra integral equation

$$z(t) = F(t) + \int_a^t k_1(t, s, z(s))\,\Delta s, \quad a, t \in I_{\mathbb{T}} = [a, +\infty) \cap \mathbb{T}, \tag{1}$$

where $z\colon I_{\mathbb{T}} \to \mathbb{R}^{2n}$ is the unknown function, and $k_1\colon I_{\mathbb{T}} \times I_{\mathbb{T}} \times \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ be rd-continuous in its first and second variable. Equation (1) is known as a Volterra integral equation on time scales. Let $F\colon I_{\mathbb{T}} \to \mathbb{R}^{2n}$, $L : I_{\mathbb{T}} \to \mathbb{R}$ be rd-continuous, $\gamma > 1$ and $\beta = L(s)\gamma$. If

$$|k_1(t, s, p) - k_1(t, s, q)| \le L(s)|p - q|, (p, q) \in \mathbb{R}^{2n}, \quad s < t, \tag{2}$$

$$m = \sup_{t \in I_{\mathbb{T}}} \frac{1}{e_\beta(t, a)} \left| F(t) + \int_a^t k_1(t, s, 0)\Delta s \right| < \infty, \tag{3}$$

then the integral equation (1) has a unique solution $z \in C^1_\beta(I_{\mathbb{T}}; \mathbb{R}^{2n})$. Let $C^1_\beta(I_{\mathbb{T}}; \mathbb{R}^{2n})$ be the linear space of rd-continuous functions such that

$$\sup_{t \in I_{\mathbb{T}}} \frac{\max(|x(t), |x^{\Delta}(t)|)}{e_\beta(t, a)} < \infty.$$

We find sufficient conditions for the existence and uniqueness of solution of integral equation (1) in $C^1_\beta(I_{\mathbb{T}}; \mathbb{R}^{2n})$.

REFERENCES

[1] A. Reinfelds, S. Christian. Nonlinear Volterra integrodifferential equations from above on unbounded time scales. *Mathematics, MDPI*, **11** (7): 1760, 2023.

[2] B. G. Pachpatte. Implict type Volterra integrodifferential equation. *Tamkang J. Math.*, **41** (1):97–107, 2010.

[3] M. Bohner, A. Peterson. *Dynamic Equations on Time Scales. An Introduction with Applications.* Birkhäuser, Boston, Basel, Berlin, 2001.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON DISCRETE APPROXIMATION BY THE LERCH ZETA-FUNCTION

AUDRONĖ RIMKEVIČIENĖ[1], DARIUS ŠIAUČIŪNAS[2]

[1]*Šiauliai State University of Applied Sciences*

Aušros av. 40, LT-76241 Šiauliai, Lithuania

E-mail: a.rimkeviciene@svako.lt

[2]*Šiauliai Academy, Vilnius University*

P. Višinskio street 25, LT-76351 Šiauliai, Lithuania

E-mail: darius.siauciunas@sa.vu.lt

The Lerch zeta-function $L(\lambda, \alpha, s)$, $s = \sigma + it$, with parameters $0 < \alpha \leqslant 1$ and $\lambda \in \mathbb{R}$ is defined for $\sigma > 1$, by Dirichlet series

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{\mathrm{e}^{2\pi i \lambda m}}{(m+\alpha)^s},$$

and is meromorphically continued to the whole complex plane. It is well known that the function $L(\lambda, \alpha, s)$ for some parameters λ and α is universal in the Voronin sense, i. e., its shifts $L(\lambda, \alpha, s+i\tau)$, $\tau \in \mathbb{R}$, approximate a wide class of analytic functions. For example, this is true for transcendental α. The problem arises does the function $L(\lambda, \alpha, s)$ have some approximation properties for all parameters λ and α. In the case of continuous shifts, this was done in [1]. In the report, we consider the approximation of analytic functions by discrete shifts $L(\lambda, \alpha, s + ikh)$, $k \in \mathbb{N} \cup \{0\}$, $h > 0$. Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$, and $H(D)$ denote the space of analytic on D functions. Our main result is the following theorem [2].

THEOREM 1. *Suppose that the parameters λ and α, and the number $h > 0$ are arbitrary. Let K be a compact set f the strip D. Then there exists a closed non-empty set $F_{\lambda,\alpha,h} \subset H(D)$ such that, for $f(s) \in F_{\lambda,\alpha,h}$ and every $\varepsilon > 0$,*

$$\liminf_{N\to\infty} \frac{1}{N+1} \# \left\{ 0 \leqslant k \leqslant N : \sup_{s\in K} |L(\lambda, \alpha, s + ikh) - f(s)| < \varepsilon \right\} > 0.$$

Moreover, the limit

$$\lim_{N\to\infty} \frac{1}{N+1} \# \left\{ 0 \leqslant k \leqslant N : \sup_{s\in K} |L(\lambda, \alpha, s + ikh) - f(s)| < \varepsilon \right\}$$

exists and is positive for all but at most countably many $\varepsilon > 0$.

REFERENCES

[1] A. Laurinčikas. "Almost" universality of the Lerch zeta-function. *Math. Commun.*, **24** (1):107–118, 2019.

[2] A. Rimkevičienė, D. Šiaučiūnas. On discrete approximation of analytic functions by shifts of the Lerch zeta-function. *Mathematics*, **10** (24):art. no. 4650, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

SCATTERING OF ELECTROMAGENTIC WAVES WITH SINGLE MEASUREMENT

SADIA SADIQUE

Tallinn University of Technology

Ehitajate tee 5, 12616 Tallinn

E-mail: sadia.sadique@ttu.ee

The study of electromagnetic waves plays a major role in the inverse scattering theory and antennas. In this research we considering the scattering of fixed frequency electromagnetic from a perfectly conducting flat screen. Fundamentally, this paper is a generalization of [1] for time harmonic Maxwell's equations in the exterior of screen. More accurately, our obstacle is two-dimensional flat screen and interacting incident wave in to three-dimensional space. This type of large and thin object model is admissible in to the study of antennas. Our main results is unique determination of both the supporting hyperplane as well as screen itself, corresponding to the single measurement having non-vanishing electric far-field. The methods we used are based on an analysis of a tangential surface integral equation on the screen. These results hold for arbitrary C^k–screens, $k = 1, 2, 3....\infty$, when they are flat, i.e included in a hyperplane.

REFERENCES

[1] E. Blästen, L. Päivärinta, S. Sadique. Unique determination of the shape of a scattering screen from a passive measurement. *Mathematics, MDPI*, **8** (7): 1156, 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON SOME FUČÍK PROBLEM WITH ONE BITSADZE-SAMARSKII TYPE NONLOCAL BOUNDARY CONDITION

NATALIJA SERGEJEVA[1], SIGITA URBONIENĖ[2]

[1] *Latvia University of Life Sciences and Technologies*
Liela street 2, Jelgava LV-3001, Latvia
[2] *Vytautas Magnus University*
Universiteto street 10, Akademija, Kaunas region LT-46265, Lithuania
E-mail: `natalija.sergejeva@lbtu.lv, sigita.urboniene@vdu.lt`

Consider the Fučík equation with one Bitsadze-Samarskii type nonlocal boundary condition

$$-x'' = \mu x^+ - \lambda x^-, \tag{1}$$
$$x(0) = 0,\ x(\xi) = 0, \tag{2}$$
$$x(\eta) = 0,\ x(1) = 0 \tag{3}$$

with the parameters $\mu, \lambda \in \mathbb{R}$ and $\xi \in (0, 1]$, $\eta \in [0, 1)$.

The spectrum of the problems (1), (2) and (1), (3) for some ξ and η values is investigated.

The idea of boundary conditions (2) and (3) was taken from the work [1], where the Sturm-Liouville equation

$$-x'' + q(t)x = \lambda^2 x$$

was analyzed (the non-negative real function $q(t)$ has a second piecewise integrable derivatives on the considered interval and λ is spectral parameter).

REFERENCES

[1] A.M.A. EL-Sayed, Z.F.A. EL-Raheem, N.A.O. Buhalima. Eigenvalues and eigenfunctions of non-local boundary value problems of the Sturm-Liouville equation. *Electronic Journal of Mathematical Analysis and Applications*, **5** (1):179–186, 2017.

[2] A. Kufner, S. Fučík. *Nonlinear Differential Equations.* Elsevier, Amsterdam, 1980.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

APPROXIMATION BY SHIFTS OF DIRICHLET L-FUNCTIONS INVOLVING GRAM FUNCTION

DARIUS ŠIAUČIŪNAS

Šiauliai Academy, Vilnius University

P. Višinskio street 25, LT-76351 Šiauliai, Lithuania

E-mail: `darius.siauciunas@sa.vu.lt`

Let χ be a Dirichlet character modulo $q \in \mathbb{N}$. The Dirichlet L-function $L(s,\chi)$, $s = \sigma + it$, is defined, for $\sigma > 1$, by

$$L(s,\chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s} = \prod_p \left(1 - \frac{\chi(p)}{p^s}\right)^{-1},$$

where the product is taken over all prime numbers, and analytic continuation elsewhere.

Let t_τ, $\tau > 0$, be the solution of the equation $\theta(t) = (\tau - 1)\pi$, where $\theta(t)$, $t > 0$, is the increment of the argument of the function $\pi^{-s/2}\Gamma(s/2)$ along the segment connecting the points $s = 1/2$ and $s = 1/2 + it$. t_τ is called the Gram function.

In the report, we consider joint approximation of analytic functions by shifts $L(s + it_\tau^\alpha, \chi)$ of Dirichlet L-functions. Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$. Denote by $\mathcal{K}$ the class of compact subsets of the strip D with connected complements, and by $H_0(K)$ the class of continuous non-vanishing functions on K that are analytic in the interior of K. The main result is the following universality theorem.

THEOREM 1. [1]. *Suppose that $\alpha_1, \dots, \alpha_r$ are fixed different positive numbers, and $\chi_1, \dots, \chi_r$ are arbitrary Dirichlet characters. For $j = 1, \dots, r$, let $K_j \in \mathcal{K}$ and $f_j(s) \in H_0(K_j)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |L(s + it_\tau^{\alpha_j}, \chi_j) - f_j(s)| < \varepsilon \right\} > 0.$$

Moreover, the limit

$$\lim_{T\to\infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |L(s + it_\tau^{\alpha_j}, \chi_j) - f_j(s)| < \varepsilon \right\}$$

exists and is positive for all but at most countably many $\varepsilon > 0$.

Also, approximation by some compositions $\Phi(L(s+it_\tau^{\alpha_1}, \chi_1), \dots, L(s+it_\tau^{\alpha_r}, \chi_r))$ will be discussed.

REFERENCES

[1] A. Laurinčikas, D. Šiaučiūnas. Joint approximation by non-linear shifts of Dirichlet L-functions. *Journal of Mathematical Analysis and Applications*, **516** (2):art. no. 126524, 2022.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

EXISTENCE OF MULTIPLE POSITIVE SOLUTIONS FOR A THIRD-ORDER BOUNDARY VALUE PROBLEM WITH NONLOCAL CONDITIONS

SERGEY SMIRNOV

Faculty of Physics, Mathematics and Optometry, University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

Institute of Mathematics and Computer Science, University of Latvia

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `sergejs.smirnovs@lu.lv`

We study the existence of multiple positive solutions for a nonlinear third-order differential equation subject to various nonlocal boundary conditions. The boundary conditions that we study, contain Stieltjes integral and include the special cases of m-point conditions and integral conditions. The main tool in the proof of our result is Krasnosel'skii's fixed point theorem. To illustrate the applicability of the obtained results, we consider examples.

REFERENCES

[1] J.R.L. Webb. Higher order non-local $(n-1,1)$ conjugate type boundary value problems. In: *Mathematical Models In Engineering, Biology And Medicine. Series: AIP Conference Proceedings. American Institute of Physics: Melville, NY*, 1124, 332–341, 2009.

[2] J.R.L. Webb, G. Infante. Non-local boundary value problems of arbitrary order. *J. London Math. Soc. (2)*, **79** (1):238–258, 2009.

[3] J.R.L. Webb, G. Infante. Positive solutions of nonlocal boundary value problems involving integral conditions. *NoDEA Nonlinear Differential Equations Appl.*, **15** (1-2):45–67, 2008.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

BIPOLAR LATTICES AS CODOMAINS FOR FUZZY EXTENSIONS OF L-TOPOLOGIES

ALEKSANDER ŠOSTAK, INGRĪDA UĻJANE

Department of Mathematics, University of Latvia

Jelgavas street 3, Riga, LV-1004, Latvia

Institute of Mathematics and Computer Science, University of Latvia

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: aleksandrs.sostaks@lu.lv, ingrida.uljane@lu.lv

Recall that given a complete lattice $(L, \leq, \wedge, \vee)$ with lower $\mathbf{0}$ and upper $\mathbf{1}$ bounds, an L-topology on a set X is a family $\tau \subseteq L^X$ containing constant functions $\mathbf{0}_X$, $\mathbf{1}_X$, and closed under taking finite meets and arbitrary joins [1],[2]. Given a complete lattice $(M, \leq_M, \wedge_M, \vee_M, \mathbf{0}_M, \mathbf{1}_M)$ (probably different from L, an (L, M)-fuzzy topology on a set X is a mapping $\mathcal{T} : L^X \to M$ such that $\mathcal{T}(\mathbf{0}_X) = \mathcal{T}(\mathbf{1}_X) = \mathbf{1}_M$, $\mathcal{T}(A \wedge B) \geq A \wedge B$ for every $A, B \in L^X$ and $\mathcal{T}(\bigvee_i A_i) \geq \bigwedge_i \mathcal{T}(A_i)$ for every $\{A_i \mid i \in I\} \subseteq L^X$ [3], [4].

Let $\mathcal{L} = L^+ \times L^-$ where $L^+ = L$ and L^- is a copy of L with reversed order, i.e. $a \leq b, a, b \in L^+ \Longrightarrow -a \geq -b, -a, -b \in L^-$. By setting $(a, b) \preceq (a', b') \Longleftrightarrow a \leq a', b \geq b'$ we introduce a partial order $\preceq$ on $\mathcal{L}$. In the result $(\mathcal{L}, \preceq)$ becomes a complete lattice that we interpret as a bipolar lattice with lower $(\mathbf{0}, -\mathbf{1})$ and upper $(\mathbf{1}, -\mathbf{0})$ bounds and with the "zero element" $(\mathbf{0}, -\mathbf{0})$.

In this talk we present a model allowing to extend an L-topology $\tau \subseteq L^X$ to a full-bodied $(L, \mathcal{L})$-fuzzy topology $\mathcal{T} = (\mathcal{O}^+, \mathcal{O}^-) : L^X \to \mathcal{L}$ where $\mathcal{O}^+(A) \in L^+$ determines the degree of openness of a fuzzy set A and $\mathcal{O}^-(A) \in L^-$ the degree of its non-openness. This model is based on the possible additional use of a binary conjunction-type operation $* : L \times L \to L$ (in particular $*$ may be lower semi-continuous t-norm or just $* = \wedge$, see [5]).The choice of the operation $*$ allows to specify different additional properties of mappings $\mathcal{O}^+ : L^X \to L^+$ and $\mathcal{O}^- : L^X \to L^-$. In particular, if $*$ is the Łukasiewicz t-norm, then $\mathcal{O}^-(A) = -\mathcal{O}^(A)$ for each $A \in L^X$ and this well corresponds with the intuitive interpretation of existence and not existence degrees of a certain property (in our case, the degrees of a fuzzy set to be open).

In conclusion, we shall sketch the ideas of application of such models for the study of other issues of fuzzy topology as well as the prospects for similar, based on bipolar lattices models for the study of other different fuzzy mathematical structures.

REFERENCES

[1] C.L. Chang. Fuzzy topological spaces. *J. Math. Anal. Appl.*, **24** :182–190, 1968.

[2] J.A. Goguen. The fuzzy Tychonoff theorem. *J. Math. Anal. Appl.*, **43** :734-742, 1973.

[3] A. Šostak. Basic structures of fuzzy topology. *J. Math. Sciences*, **78** (6):662-701, 1996.

[4] T. Kubiak, A. Šostak. A fuzzification of the category of M-valued L-topological space. *Applied General Topology*, **5** (2):137-154, 2004.

[5] E.P. Klement, R. Mesiar, E. Pap. *Triangular Norms*. Kluwer Publ., 2000.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ASYMPTOTIC ANALYSIS OF STURM–LIOUVILLE PROBLEM WITH NONLOCAL BOUNDARY CONDITIONS

ARTŪRAS ŠTIKONAS

Institute of Apllied Mathematics, Vilnius University

Naugarduko street 24, LT-03225, Vilnius, Lithuania

E-mail: `arturas.stikonas@mif.vu.lt`

Consider the following one-dimensional Sturm–Liouville equation

$$-u''(t) + q(t)u(t) = \lambda u(t), \quad t \in [0,1], \tag{1}$$

where the real-valued function $q \in C[0,1]$.

Sturm–Liouville Problem (SLP) which consist of equation (1) with one classical (local) boundary condition $u(0) = 0$ [3] or $u'(0) = 0$ [1] and another two-point Nonlocal Boundary Condition (NBC)

$$\text{(Case 1)} \qquad u'(1) = \gamma u(\xi), \qquad \xi \in [0,1], \tag{2_1}$$

$$\text{(Case 2)} \qquad u'(1) = \gamma u'(\xi), \qquad \xi \in [0,1), \tag{2_2}$$

$$\text{(Case 3)} \qquad u(1) = \gamma u(\xi), \qquad \xi \in [0,1), \tag{2_3}$$

where $\gamma \in \mathbb{R}$ were considered.

We obtained asymptotic formulas for eigenvalues and eigenfunctions for these two cases. Now we investigate more general case with Robin type boundary condition $\cos\alpha\, u(0) + \sin\alpha\, u'(0) = 0$, $\alpha \in (0,\pi)$ [2] and NBC (2).

Finally we discussed how to get asymptotic expansions for real eigenvalues in the case more complicated NBC.

REFERENCES

[1] A. Štikonas, E. Şen. Asymptotic analysis of Sturm–Liouville problem with Neumann and nonlocal two-point boundary conditions. *Lith. Math. J.*, **62** :519–541, 2022. https://doi.org/10.1007/s10986-022-09577-6

[2] A. Štikonas. Asymptotic analysis of Sturm–Liouville problem with Robin and two-point boundary conditions. *Liet. matem. rink. Proc. LMS,* Ser. A, **63**:9–18, 2022. https://doi.org/10.15388/LMR.2022.29692

[3] A. Štikonas, E. Şen. Asymptotic analysis of Sturm–Liouville problem with Dirichlet and nonlocal two-point boundary conditions. *Math. Model. Anal.*, **28** (2):308–329, 2023. https://doi.org/10.3846/mma.2023.17617

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

FINITE DIFFERENCE METHOD FOR TWO-DIMENSIONAL ELLIPTIC EQUATION WITH THE MULTIPLE INTEGRAL IN NONLOCAL CONDITION

OLGA ŠTIKONIENĖ[1], MIFODIJUS SAPAGOVAS[2]

[1]*Institute of Applied Mathematics, Vilnius University*
Naugarduko 24, LT-03225, Vilnius, Lithuania
[2]*Institute of Data Science and Digital Technologies, Vilnius University*
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: `olga.stikoniene@mif.vu.lt, mifodijus.sapagovas@mif.vu.lt`

We consider finite difference approximation to the solution of following boundary value problem

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y,u), \quad (x,y) \in \Omega = \{0 < x, y < 1\}, \\
&u|_{\partial\Omega} = \iint_{\Omega} K(x,y)u(x,y)dxdy.
\end{aligned}
$$

The main aims of our study are the convergence of finite difference scheme and the iterative methods for solution of the system of nonlinear difference equations.

Motivated by some theoretical results of [1] and [2] we study the spectrum structure of the corresponding difference eigenvalue problem.

REFERENCES

[1] Y. Lin, S. Xu. Finite difference approximation for a class of non-local parabolic equations. *Internat. J. Math. and Math. Sci.*, **20** (1):147–164, 1997.

[2] Y. Wang. Solutions to nonlinear elliptic equations with a nonlocal boundary condition. *Electronic Journal of Differential Equations*, **2002** (05):1–16, 2002.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

AN APPLICATION OF SPH TO COMBUSTION

ULDIS STRAUTIŅŠ, ALEKSANDRS LONČS

University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: `uldis.strautins@lu.lv, alloncs@inbox.lv`

We present a simulation technique for the system of compressible Navier-Stokes equations, barotropic gas pressure-density law, heat equation and convection-reaction-diffusion equations with Arrhenius kinetics in Lagrange coordinates:

$$T_t = D\triangle T + c_1 c_2 Q e^{-E/k_B T},$$

$$c_{i,t} = D_i \triangle c_i - c_1 c_2 k e^{-E/k_B T}, \quad i \in \{1, 2\},$$

$$v_t = -\frac{1}{\rho}\nabla P + \nu\triangle v + \beta(T - T_0),$$

$$P = P_0 \rho^\alpha, \quad \rho_t = -\mathrm{div}(\rho v),$$

where T is temperature field, c_1 is the concentration of fuel, c_2 the concentration of oxygen, v is the gas velocity field, P is pressure field, ρ density, D is the heat diffusion coefficient, D_1 and D_2 the diffusion coefficients of the fuel and oxygen, ν viscosity, β thermal expansion coefficient.

We develop and implement a smoothed particle hydrodynamics (SPH) method [1] for this system of equations with various boundary conditions. The kernel function is a Gauss kernel

$$W(x, h) = (h\sqrt{\pi})^{-d} exp(-x^2/h^2),$$

cut off at $|x| = 3h$, so the Laplace operator is approximated as

$$\triangle f(X_i) \approx 2\sum_j \frac{m_j}{\rho_j} f(X_j) \frac{X_i - X_j}{|X_i - X_j|^2} \nabla W(X_i - X_j, h).$$

REFERENCES

[1] D. Violeau. *Fluid Mechanics and the SPH Method.* Oxford University Press, 2012.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON MATHEMATICAL MODELLING, SIMULATION AND OPTIMIZATION OF SUPERCRITICAL CO2 EXTRACTION FLOW

ULDIS STRAUTIŅŠ[1,2], MAKSIMS MARINAKI[1,2], MANFREDS ŠNEPS-ŠNEPPE[3], HARIJS KALIS[1]

[1] *Institute of Mathematics and Computer Science, University of Latvia*

Raina blvd. 29, Riga LV-1459, Latvia

[2] *Department of Mathematics, University of Latvia*

Jelgavas street 3, Riga LV-1004, Latvia

[3] *Ventspils University of Applied Sciences*

Inzenieru street 101, Ventspils, LV-3601, Latvija

E-mail: `uldis.strautins@lu.lv`

E-mail: `maksims.marinaki@lu.lv`

E-mail: `manfreds.sneps@venta.lv`

E-mail: `harijs.kalis@lu.lv`

Supercritical carbon dioxide extraction is a process of extracting valuable compounds from a wide range of materials using CO2 in its supercritical state. Authors consider mathematical models for 3D flow in extraction of biologically active compounds from plants. Similar models and treatment of experimental data for the process also considered in [1], [2].

The governing equations are

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) = -\nabla p + \frac{1}{Re} \triangle \mathbf{v} + \mathbf{f},$$

$$\frac{\partial \varphi}{\partial t} + \nabla \cdot (\mathbf{v}\varphi) = \alpha \triangle \varphi + \mathbf{R}(\varphi, \psi),$$

where $\varphi(r,t)$ is the concentration of the active compound in the solvent and $\psi(r,t)$ is the concentration of the active compound bounded in the intact cells.

These are discretized by finite volume techniques. We discuss parametrization of the process as well and some techniques how to perform the optimization process. Moreover we present the graphical results of obtained fields and extraction curves and discuss main aspects of several underlying modelling stages.

REFERENCES

[1] H. Sovová. Mathematical model for supercritical fluid extraction of natural products and extraction curve evaluation. *J. of Supercritical Fluids*, **33** :35–52, 2005.

[2] E.L.G. Oliveira, A.J.D. Silvestre, C.M. Silva. Review of kinetic models for supercritical fluid extraction. *Chemical Engineering Research and Design*, **89** :1104–1117, 2011.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

CITATION INDICES DEPENDENT ON TIME

ANDREA STUPŇANOVÁ

Slovak University of Technology in Bratislava, Faculty of Civil Engineering

Radlinského 11, Bratislava, Slovak Republic

E-mail: andrea.stupnanova@stuba.sk

Citation analysis is a globally used tool for evaluating the scientific performance and impact of various subjects, including scholars (scientists, researchers), journals, etc. Several citation indices have been introduced for this purpose. Certainly the most considered index in this area is the Hirsch index H, see [2]. Almost immediately after the introduction of the h-index H, several complaints appeared. Subsequently, a number of its modifications were published. Among the other used citation indices, let's us mention g-index of Egghe G, see [1] and Kosmulsky's MAXPROD index, [3]. Most citation indices (including those already mentioned ones) do not distinguish the publication time of papers of the considered author and are based on a simple characterization of each author by a decreasing vector $\mathbf{a}$ only. Note that the vector of citation is defined as $\mathbf{a} = (a_1, \ldots, a_n)$, $a_1 \geq \cdots \geq a_n$, where n is the number of published papers of author A (in the considered database) and a_i denotes the number of citations obtained by i- th most cited article. Then scholars who have no papers or citations in recent decades will retain their (possibly high) (static) citation index.

Note that there were several attempts to introduce time-dependent (generalization of) h-index. We will use a different approach when describing the time-dependent author's performance, where not only the total number of citations of the involved papers is taken into account (the vector $\mathbf{a}$), but also the time of publication, where a vector $\mathbf{y} = (y_1, \ldots, y_n) \in \mathbb{N}^n$ is considered. Here y_i is the year age of the i-th paper possessing a_i citations.

Consequently, we exploit the fact that the h-index and MAXPROD can be considered as Sugeno and Shilkret integrals with respect to the counting measure, respectively. Note that the g-index cannot be represented by some integral. To introduce new time-dependent (dynamic) indices, we propose 3 ways to modify the mentioned citation indices. In the first one, we replace the input vector $\mathbf{a}$ by an reordered vector $\left(\frac{a_1}{y_1}, \ldots, \frac{a_n}{y_n}\right)$ (this approach can be applied to arbitrary indices, not only represented by an integral), in the second one we change the counting measure to a measure dependent on the vector $\mathbf{y}$, and finally we combine the two previous ones.

We believe that our approach is more fair for the scientific beginners and for all scholars active in the field. More, our new dynamic indices allow to compare authors with the same or similar static indices in a way where active scientists are ranked higher. We will show the advantages of these new dynamic indexes on examples.

Acknowledgment. The work was supported from grants VEGA1/0468/20 and VEGA1/0036/23.

REFERENCES

[1] L. Egghe. Theory and practise of the g-index. *Scientometrics*, **69** :131–152, 2006.

[2] J.E. Hirsch. An index to quantify an individual's scientific research output. *Proc. Nat. Acad. Sci.*, **102** (4):16569–16572, 2005.

[3] M. Kosmulski. MAXPROD – A new index for assessment of the scientific output of an individual, and a comparison with the h index. *Cybermetrics*, **11** (1): paper 5, 2007.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON UNIVERSALITY OF PERIODIC ZETA-FUNCTIONS

MONIKA TEKORĖ[1], DARIUS ŠIAUČIŪNAS[2]

[1] *Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: monika.tekore@mif.stud.vu.lt

[2] *Šiauliai Academy, Vilnius University*

P. Višinskio street 25, LT-76351 Šiauliai, Lithuania

E-mail: darius.siauciunas@sa.vu.lt

Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ be a periodic with period q sequence of complex numbers, and $s = \sigma + it$ a complex variable. The periodic zeta-function $\zeta(s; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series $\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} a_m m^{-s}$, and has a meromorphic continuation to the whole complex plane.

Suppose that the sequence $\mathfrak{a}$ additionally is multiplicative, i. e., $a_{mn} = a_m a_n$ for all $(m, n) = 1$, and $a_1 = 1$. For example, a Dirichlet character modulo q is multiplicative. Then it is known [1] that the function $\zeta(s; \mathfrak{a})$ is universal in the sense that the shifts of $\zeta(s + i\tau; \mathfrak{a})$, $\tau \in \mathbb{R}$, approximate a wide class of analytic functions.

In the report, we consider the joint universality of periodic zeta-functions with multiplicative coefficients. Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$, $\mathcal{K}$ be the class of compact sets of the strip D with connected complements, and $H_0(K)$, $K \in \mathcal{K}$, the class of continuous non-vanishing functions on K that are analytic in the interior of K. Denote by $U_1(T_0)$ the class of real increasing to $+\infty$ continuously differentiable functions $\gamma(\tau)$ with monotonic derivative $\gamma'(\tau)$ on (T_0, ∞) such that $\gamma(2\tau) \max_{\tau \leqslant u \leqslant 2\tau} (\gamma'(u)^{-1}) \ll \tau$ as $\tau \to \infty$. Then we have

Theorem 1. [2]. *Suppose that the sequences $\mathfrak{a}_1, \ldots, \mathfrak{a}_r$ multiplicative, $a_1, \ldots, a_r$ are real algebraic numbers linearly independent over $\mathbb{Q}$, and $\gamma(\tau) \in U_1(T_0)$. For $j = 1, \ldots, r$, let $K_j \in \mathcal{K}$ and $f_j(s) \in H_0(K_j)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T \to \infty} \frac{1}{T} \operatorname{meas} \left\{ \tau \in [0, T] : \sup_{1 \leqslant j \leqslant r} \sup_{s \in K_j} |\zeta(s + i a_j \gamma(\tau); \mathfrak{a}_j) - f_j(s)| < \varepsilon \right\} > 0.$$

Also, the joint approximation by shifts $(\zeta(s + i\gamma_1(\tau); \mathfrak{a}_1), \ldots, \zeta(s + i\gamma_r(\tau); \mathfrak{a}_r))$ will be considered. In this case, we require that the functions $\gamma_j(\tau)$ satisfy $\gamma_j(\tau) = \widehat{\gamma}_j(\tau)(1 + o(1))$, where $\widehat{\gamma}_j(\tau) = o(\widehat{\gamma}_{j_0}(\tau))$ for some j_0.

Moreover, we discuss the joint discrete approximation of analytic functions by shifts $(\zeta(s + ih_1\gamma_1; \mathfrak{a}_1), \ldots, \zeta(s + ih_r\gamma_r; \mathfrak{a}_r))$, where $\{\gamma_k : k \in \mathbb{N}\}$ is the sequence of imaginary parts $\gamma_k > 0$ of non-trivial zeros of the Riemann zeta-function.

REFERENCES

[1] A. Laurinčikas, D. Šiaučiūnas. Remarks on the universality of the periodic zeta-function. *Mathematical Notes*, **80** (1-2):532–538, 2006.

[2] A. Laurinčikas, M. Tekorė. Joint universality of periodic zeta-functions with multiplicative coefficients. *Nonlinear Analysis: Modelling and Control*, **25** (5):860–883, 2020.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

CONSTRUCTION OF SOLUTIONS TO CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS VIA OPERATOR TECHNIQUES

INGA TELKSNIENĖ, ZENONAS NAVICKAS, MINVYDAS RAGULSKIS

Center for Nonlinear Systems, Kaunas University of Technology

Studentu 50-147, Kaunas LT-51368, Lithuania

E-mail: `inga.telksniene@ktu.lt`

Fractional differential equations (FDEs) have emerged in recent decades as an important class of models that can be used to describe various real-world phenomena. These equations have been successfully used in a plethora of fields ranging from physical applications [1], health research [2] and even finance [3].

In this study, the solutions y to FDEs are represented as power series with fractional basis elements $y = \sum_{j=0}^{+\infty} c_j x^{j/n}$, where the fractional differentiation order is $\alpha = \frac{1}{n}, n \in \mathbb{N}$. This concept of solutions to FDEs allows the consideration of equivalency between FDEs of a specific form and respective ordinary differential equations (ODEs) that are obtained via a nonlinear independent variable transformation. However, the aforementioned transformation can only lead to a computational scheme if a certain transfer function is approximated via a series truncation.

Mathematically, the scheme can be described as follows. Consider the following FDE:

$$^{C}\mathbf{D}^{(1/n)} y = F(x, y),$$

where $F(x, y)$ is an arbitrary function. Firstly, this FDE is transformed into:

$$\left(^{C}\mathbf{D}^{(1/n)} \right)^n y = G(x, y) + \Psi(x),$$

where $G(x, y)$ and $\Psi(x)$ depend on the form of $F(x, y)$. The function $\Psi(x)$ is given by power series with coefficients defined via a recurrence relation. Finally, the transformation $t = \sqrt[n]{x}$ yields:

$$\frac{\mathrm{d}\widehat{y}}{\mathrm{d}t} = H\left(\widehat{y}, t\right), \quad y = y(x) = \widehat{y}\left(\sqrt[n]{x}\right).$$

This ODE can be analyzed via conventional techniques and its solution transformed into the FDE solution using the inverse transformation.

REFERENCES

[1] R. Hilfer. *Applications of Fractional Calculus in Physics.* World Scientific, Singapore, 2000.

[2] D. Kumar, J. Singh. *Fractional Calculus in Medical and Health Science.* CRC Press, Boca Raton, US, 2020.

[3] V. Tarasov, V. Tarasova. *Economic Dynamics with Memory: Fractional Calculus Approach.* Walter de Gruyter, Berlin, Germany, 2021.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

SPECTRUM PROPERTIES OF THE FUČÍK PROBLEM WITH NONLOCAL BOUNDARY CONDITION

SIGITA URBONIENĖ[1], NATALIJA SERGEJEVA[2]

[1] *Vytautas Magnus University*

Universiteto street 10, Akademija, Kaunas region LT-46265, Lithuania

[2] *Latvia University of Life Sciences and Technologies*

Liela street 2, Jelgava LV-3001, Latvia

E-mail: `sigita.urboniene@vdu.lt, natalija.sergejeva@lbtu.lv`

Let us analyze the Fučík problem with nonlocal two-point condition in the right side boundary

$$x'' = -\mu x^+ + \lambda x^-, \tag{1}$$

$$x(0) = 0, \tag{2}$$

$$x(1) = \gamma x(\xi), \quad \gamma \in \mathbb{R}, \tag{3}$$

here $\xi = \frac{m}{n} \in (0,1)$, m and n $(0 < m < n)$ are positive coprime integer numbers. The spectrum of the Fučćk problem with classical boundary conditions are quite well analyzed. However, in case of nonlocal boundary conditions the spectrum can differ considerably. Thus, the aim of investigation is to analyze main properties of the nonlocal problem spectrum and compare them with classical Fučćk spectrum. The analytical description and visualization of the spectrum will be provided.

Some of the results are the logical continuation and generalization of previous authors' investigations. This investigation continues and generalize the results obtained for problem (1) – (3) with $\xi = \frac{1}{2}$ [1], $\xi = \frac{1}{3}$ [2], $\xi = \frac{1}{4}$ [3] and $\xi = \frac{1}{n}$ [4].

REFERENCES

[1] N. Sergejeva. The regions of solvability for some three point problem. *Math. Model. Anal.*, **18** (2):191–203, 2013.

[2] E. Kidikaitė, S. Pečiulytė, A. Štikonas. Fučík spectrum for problem with nonlocal two-point boundary condition. In: *18th International Conference on Mathematical Modelling and Analysis (MMA 2013)*, 47, 2013.

[3] N. Sergejeva. The Fučík spectrum for some boundary value problem. In: *Proc. of IMCS of University of Latvia*, 14, 65–75, 2014.

[4] N. Sergejeva. On some Fučík type problem with nonlocal boundary condition. In: *Proc. of IMCS of University of Latvia*, 19, 57–64, 2019.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ADVANTAGES OF THE SMOOTHLY TRIMMED MEAN

JANIS VALEINIS

University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: janis.valeinis@lu.lv

L-estimators, defined as a linear combination of order statistics, are commonly used for robust inference of the central tendency. The trimmed mean is one of the most popular L-estimators, among others. In 1973 M. Stigler [1] showed that it has a significant disadvantage the limiting distribution of the trimmed mean may not be asymptotically normal if the data have a discrete or continuous distribution with gaps. In his paper M. Stigler proposed to use the smoothly trimmed and showed that the asymptotic distribution of such statistics is asymptotically normal for any distribution.

We propose a slightly more general version of smoothly trimmed mean, derive its variance and establish the empirical likelihood method for this estimator. There are many statistical procedures which are based on the classical trimmed mean estimator (see, for example, the classical book by Wilcox [2]). In this work for different testing procedures we simply replace the classical trimmed mean estimator by the smoothly trimmed mean estimator and compare them by extensive simulation analysis. Some applications for real data examples will be analysed as well.

REFERENCES

[1] S.M. Stigler. The asymptotic distribution of the trimmed mean. *The Annals of Statistics*, **1** (3):472–477, 1973.

[2] R.R. Wilcox. *Introduction to Robust Estimation and Hypothesis Testing.* Academic Press (4th edition) Elsevier, Burlington, MA, 2017.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ON JOINT UNIVERSALITY IN THE SELBERG-STEUDING CLASS

BRIGITA ŽEMAITIENĖ

Institute of Mathematics, Faculty of Mathematics and Informatics, Vilnius University

Naugarduko street 24, LT-03225 Vilnius, Lithuania

E-mail: `brigita.zemaitiene@mif.vu.lt`

The Selberg class $\mathcal{S}$ is defined axiomatically and consists of Dirichlet series satisfying four following axioms: the Ramanujan hypothesis, an analytic continuation, functional equation, multiplicativity. The Selberg-Steuding class $\mathcal{S}^*$ is a complemented Selberg class by an arithmetic hypothesis related to distribution of prime numbers and an existence of the Euler product.

The universality property for the Riemann zeta-function $\zeta(s)$ (which is a member of $\mathcal{S}$ also) was obtained by S.M. Voronin in [3], while the first universality result related to the class $\mathcal{S}^*$ was obtained by J. Steuding in [2].

In the report, we present the joint universality theorem for L-functions from the Selberg - Steuding class of functions belonging to the class $\mathcal{S}$ and satisfying the arithmetic condition on the distribution of primes (denote this fact $\mathcal{S}_1$). More precisely, we discus simultaneous approximations of a collection of analytic functions $\big(f_1(s), \dots, f_r(s)\big)$ in the strip $D_L = \{s \in \mathbb{C} : \sigma_L < \sigma < 1\}$ by a collection of shifts $\big(L(s+ia_1\tau), \dots, L(s+ia_r\tau)\big)$, $L(s) \in \mathcal{S}_1$, where $\sigma_L > \frac{1}{2}$ is a certain number depending on L.

Denote by $\mathcal{K}_L$ the class of compact subset of the strip D_L with connected complements, and by $H_{0L}(K)$, $K \in \mathcal{K}_L$, the class of continuous non-vanishing functions on K that are analytic in the interior of K. The main result is following theorem.

THEOREM 1. *Suppose that $L(s) \in \mathcal{S}_1$, and real algebraic numbers $a_1, \dots, a_r$ are linearly independent over the field of rational numbers $\mathbb{Q}$. For $j = 1, \dots, r$, let $K_j \in \mathcal{K}_L$ and $f_j(s) \in H_{0L}(K_j)$. Then, for every $\epsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas}\left\{\tau \in [0,T] : \sup_{1\le j\le r} \sup_{s\in K_j} |L(s+ia_j\tau) - f_j(s)| < \epsilon\right\} > 0.$$

Moreover, "$\liminf$" can be replaced by "$\lim$" for all but at most countably many $\epsilon > 0$.

The content of presentation is based on the joint work by R. Kačinskaitė, A. Laurinčikas and B. Žemaitienė [1].

REFERENCES

[1] R. Kačinskaitė, A. Laurinčikas, B. Žemaitienė. On joint universality in the Selberg-Steuding class. *Mathematics*, **11, 737** 2023. https://doi.org/10.3390/math11030737

[2] J. Steuding. Value Distribution of L-Functions *Lecture Notes Math 1877.* Springer: Berlin/Heidelberg, Germany, New York, USA, 2007.

[3] S.M. Voronin. Theorem on the "universality" of the Riemann zeta-function. *Math. USSR-Izv*, **9** :443–453, 1975.

Abstracts of MMA2023, May 30 – June 2, 2023, Jurmala, Latvia

ESTIMATION OF THE SOLUTIONS OF A MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM

MĀRTIŅŠ ZEMLĪTIS, OLGA GRIGORENKO

Institute of Mathematics and Computer Science, University of Latvia

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: martins.zemlitis@gmail.com; ol.grigorenko@gmail.com

Multi-objective linear programming (MOLP) is a type of mathematical optimization technique used to solve problems that involve multiple conflicting objectives. In MOLP, the goal is to find a set of solutions that simultaneously optimize two or more objectives, subject to a set of linear constraints. MOLP problems are used in various fields, including engineering, economics, finance, and environmental management. They are particularly useful in situations where decision makers must balance multiple objectives.

When solving multi-objective linear programming problems using Zimmermann's objective functions [2] or fuzzy orderings [1], it can be seen, that the solutions tend to be *close* to the centroid or mass center of individual solutions set. In this case, we define proximity by metrics, which shows how far the point is from the individual solution in terms of an objective function values. In this work, the observation that the optimal solutions are *around* the centroid is looked at in details. Intuitively, the question arises whether it is true that

$$d(x; c_j) \geq d(x; c_M),$$

where x is an optimal solution, c_j are individual solutions, c_M is a centroid and d is a global metric or distance-like function which takes into account all the metrics described above. Thus, this work is devoted to finding the necessary assumptions for the inequality to be true, and finding the best way to determine the point c_M.

The inequality can be used as a rough estimate of the region where the optimal solution for a given MOLP problem can be found. It is useful when the optimisation is done using an iterative algorithm. It allows to start iterations closer to the optimal solution, by choosing the point c_M or points around it as the starting points, thus expecting, that the algorithm will converge faster.

Acknowledgment. The authors are thankful for financial support European Regional Development Fund within the project Nr.1.1.1.2/16/I/001, application Nr.1.1.1.2/VIAA/4/20/707 "Fuzzy relations and fuzzy metrics for customer behavior modeling and analysis".

REFERENCES

[1] O. Grigorenko. Involving fuzzy orders for multi-objective linear programming. *Mathematical Modelling and Analysis*, **17** (3):366–382, 2012. https://doi.org/10.3846/13926292.2012.685958

[2] H. J. Zimmermann. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, **1** :45–55, 1978. https://doi.org/10.1016/0165-0114(78)90031-3

Index of Authors