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ABSTRACTS

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ON SINE AND COSINE TYPE FUNCTIONS, ARISING IN THE THEORY OF NONLINEAR DIFFERENTIAL EQUATIONS

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In the beginning of the study of some classes of nonlinear differential equations it is useful sometimes to start with the related autonomous equations and to look at the properties of solutions. For instance, the important is role playing by generalized sine and cosine functions in the theory of the so called half-linear differential equations [1]. Another instance is the lemniscatic sine and cosine functions, appearing in the Nehari theory of characteristic numbers [2].

Passing to the results, let us denote by S and C those solutions of the Emden - Fowler type differential equation $x'' = -3x^5$, which satisfy the initial conditions $x(0) = 0$, $x'(0) = 1$ and $x(0) = 1$, $x'(0) = 0$ respectively. We know the following properties of the functions S and C.

- The functions S and C are periodical with the minimal period $T = 4\int_0^1 \frac{dt}{\sqrt{1-t^6}}$, defined on entire R and with the value range $[-1; 1]$.
- The functions S and C can be represented by the Jacobian elliptic functions, namely, for any real t the following relations hold

$$S(t) = \frac{\operatorname{sn}(\alpha t; k)}{\sqrt{\alpha^2 \operatorname{cd}^2(\alpha t; k) + \operatorname{sn}^2(\alpha t; k)}}, \; C(t) = \frac{\operatorname{cd}(\alpha t; k)}{\sqrt{\operatorname{cd}^2(\alpha t; k) + \alpha^2 \operatorname{sn}^2(\alpha t; k)}}, \; \alpha = \sqrt[4]{3}, \; k = \frac{\sqrt{2-\sqrt{3}}}{2}.$$

- For any real t the following analogue of the main trigonometric identity holds

$$S^2(t) + 2S^2(t)C^2(t) + C^2(t) = 1 \quad \text{or} \quad \big(1 + 2S^2(t)\big)\big(1 + 2C^2(t)\big) = 3.$$

- For any real t the following holds

$$S'(t) = C(t)\sqrt{1 + 3S^2(t) + 3S^4(t) + 2S^6(t)}, \; C'(t) = -S(t)\sqrt{1 + 3C^2(t) + 3C^4(t) + 2C^6(t)}.$$

- The addition theorem holds for the functions S and C in the form: there exist the two-argument functions Φ and Ψ such, that for any real u and v the relation $S(u+v) = \Phi\big(S(u); S(v)\big)$ and $C(u+v) = \Psi\big(C(u); C(v)\big)$ holds. If $u = v$, then $S(2u) = \frac{2S(u)S'(u)}{\sqrt{1+8S^6(u)}}$.

REFERENCES

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