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PARALLEL BRANCH AND BOUND ALGORITHM FOR MULTIDIMENSIONAL SCALING WITH CITY-BLOCK DISTANCES

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Multidimensional scaling is a technique for exploratory analysis of multidimensional data widely usable in different applications [1]. Pairwise dissimilarities among n objects are given by the matrix (δ_{ij}) , i, j = 1, ..., n. A set of points in an embedding metric space is considered as an image of the set of objects. Normally, an m-dimensional vector space is used, and $\mathbf{x}_i \in \mathbf{R}^m$, i = 1, ..., n, should be found whose inter-point distances fit the given dissimilarities. Images of the considered objects can be found minimizing a fit criterion, e.g. the most frequently used least squares *STRESS* function:

$$S(\mathbf{x}) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \left(d\left(\mathbf{x}_{i}, \mathbf{x}_{j}\right) - \delta_{ij} \right)^{2},$$

where $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$, $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{im})$; $d(\mathbf{x}_i, \mathbf{x}_j)$ denotes the distance between the points \mathbf{x}_i and \mathbf{x}_j ; it is supposed that the weights are positive: $w_{ij} > 0, i, j = 1, \dots, n$. The most frequently used distances are Euclidean, but multidimensional scaling with other Minkowski distances in the embedding space can be even more informative. In the present paper the problem with the *STRESS* criterion and city-block distances in the embedding space are considered.

STRESS normally has many local minima. The case of city-block distances is different from the other cases of Minkowski distances that STRESS with city-block distances can be non differentiable even at a minimum point [2]. However it is piecewise quadratic, and such a structure can be exploited for tailoring of an ad hoc global optimization algorithm. A two level minimization method for the two-dimensional embedding space was proposed in [2] where a problem of combinatorial optimization is tackled by evolutionary search at the upper level, and a problem of quadratic programming is tackled at the lower level. A branch and bound algorithm for the upper level combinatorial problem is proposed in [3].

In this lecture parallel version of the branch and bound algorithm for multidimensional scaling is presented. The parallel algorithm is investigated experimentally solving geometrical and empirical data sets. The results of experiments are discussed and directions for future research are identified.

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