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CORDIAL VOLTERRA INTEGRAL EQUATIONS

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Consider the class of Volterra operators $(Vu)(t) = \int_0^t K(t,s)u(s)ds,\ 0 \le t \le T$, having the following property (A): V is a bounded operator in $C[0,T]$ and $u_r(t) = t^r,\ 0 \le r < \infty$, are eigenfunctions of V. Such an operator in noncompact in $C[0,T]$. It occurs that V has property (A) if and only if its kernel has the form $K(t,s) = t^{-1}\varphi(s/t)$, $0 \le s \le t \le T$, where $\varphi \in L^1(0,1)$. Thus we actually study the class of operators

$$(V_\varphi u)(t) = \int_0^t t^{-1}\varphi(s/t)u(s)ds = \int_0^1 \varphi(x)u(tx)dx,\ 0 \le t \le T.$$

We call $\varphi \in L^1(0,1)$ the *core* of V_φ and V_φ itself an operator with an core, or simply *cordial operator*. We introduce in $L^1(0,1)$ a multiplication operation $\varphi \star \psi$ so that $V_\varphi V_\psi = V_{\varphi\star\psi}$.. So $L^1(0,1)$ and the class of cordial operators become commutative Banach algebras which are isometrically isomorphic. In this way we establish formulae for the spectrum $\sigma_m(V_\varphi)$ of V_φ as an operator in $C^m[0,T]$:

$$\sigma_0(V_\varphi) = \{0\} \cup \{\hat\varphi(\lambda) :\ \lambda \in \mathbb{C},\ \mathrm{Re}\lambda \ge 0\},\ m = 0,$$

$$\sigma_m(V_\varphi) = \{0\} \cup \{\hat\varphi(k) :\ k = 0, ..., m-1\} \cup \{\hat\varphi(\lambda) :\ \mathrm{Re}\lambda \ge m\},\ m = 1, 2, ...,$$

where $\hat\varphi(\lambda) = \int_0^1 \varphi(s)s^\lambda ds$. Note that $\sup\{|\ \hat\varphi(\lambda)\ |:\ \mathrm{Re}\lambda \ge m\} \to 0$ as $m \to \infty$. In particular, we localise the spectra of Diogo's, Lighthill's and some other noncompact Volterra integral operators occuring in the practice.

We also treat the Volterra integral operators of a more general form

$$(V_{\varphi,a}u)(t) = \int_0^t t^{-1}\varphi(s/t)a(t,s)u(s)ds,\ \ 0 \le t \le T,$$

where $\varphi \in L^1(0,1),\ a \in C^m(0 \le s \le t \le T),\ m \ge 0$. It occurs that $\sigma_m(V_{\varphi,a}) = a(0,0)\sigma_m(V_\varphi)$. In particular, if $a(0,0) = 0$ then $\sigma_m(V_{\varphi,a}) = \{0\}$ and $V_{\varphi,a}$ as an operator in $C^m[0,T]$ occurs to be compact.

We prove the convergence and establish error estimates of the polynomial collocation methods for the Volterra integral equation $\mu u = V_{\varphi,a}u + f$ assuming that $\mu \ne 0$, $\mu \ne a(0,0)\hat\varphi(k)$, $k = 0, 1, ...$, and that $f \in C^m[0,T]$ where m is sufficiently large so that $\mu \notin a(0,0)\sigma_m(V_\varphi)$; in particular, $m = 0$ suits if $\mu \notin a(0,0)\sigma_0(V_\varphi)$, i.e., $\mu \ne a(0,0)\hat\varphi(\lambda)$ for $\mathrm{Re}\lambda \ge 0$.