

CHARACTERISTIC FUNCTIONS FOR STURM-LIOUVILLE PROBLEMS WITH NONLOCAL BOUNDARY CONDITION

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Let us consider a Sturm–Liouville problem with the NBC:

$$-(p(t)u')' + q(t)u = \lambda u, \quad t \in (0, 1), \quad (1)$$

$$\langle l_0, u(t) \rangle = 0, \quad (2)$$

$$\langle l_1, u(t) \rangle = \gamma \langle k, u(t) \rangle \quad (3)$$

where $p(t) \geq p_0 > 0$, $p \in C^1[0, 1]$, $q \in C[0, 1]$, l_0 , l_1 and k are linear functionals. For example, the functional k can describe multi-point or integral NBCs:

$$\langle k, u(t) \rangle = \sum_{j=1}^n (\varkappa_j u(\xi_j) + \kappa_j u'(\xi_j)), \quad \langle k, u(t) \rangle = \int_0^1 \varkappa(t) u(t) dt,$$

and the functional l_i , $i = 0, 1$ can describe local (classical) boundary conditions

$$\langle l_0, u(t) \rangle = \alpha_0 u(0) + \beta_0 u'(0), \quad \langle l_1, u(t) \rangle = \alpha_1 u(1) + \beta_1 u'(1),$$

where the parameters $|\alpha_i| + |\beta_i| > 0$, $i = 0, 1$.

Let $\varphi_0(t; \lambda)$ and $\varphi_1(t; \lambda)$ be two independent solutions of equation (1) and

$$D_s^t(\lambda) := \begin{vmatrix} \varphi_0(t; \lambda) & \varphi_1(t; \lambda) \\ \varphi_0(s; \lambda) & \varphi_1(s; \lambda) \end{vmatrix}, \quad \langle k_1 \cdot k_2, D_s^t(\lambda) \rangle := \begin{vmatrix} \langle k_1, \varphi_0(t; \lambda) \rangle & \langle k_1, \varphi_1(t; \lambda) \rangle \\ \langle k_2, \varphi_0(s; \lambda) \rangle & \langle k_2, \varphi_1(s; \lambda) \rangle \end{vmatrix}.$$

All the solutions of equation (1) are of the form $u = C_0 \varphi_0(t; \lambda) + C_1 \varphi_1(t; \lambda)$. There exists a nontrivial solution of problem (1)–(3) if and only if $\Psi(\lambda)\gamma = \Phi(\lambda)$, where $\Psi := \langle l_0 \cdot k, D_s^t(\lambda) \rangle$, $\Phi := \langle l_0 \cdot l_1, D_s^t(\lambda) \rangle$. Both functions $\Psi(\lambda)$ and $\Phi(\lambda)$ are entire functions for $\lambda \in \mathbb{C}$.

The complex characteristic function

$$\gamma_c := \Phi(\lambda)/\Psi(\lambda), \quad \gamma_c: \mathbb{C}_\lambda \rightarrow \overline{\mathbb{C}} \quad (4)$$

is a meromorphic function and describes complex eigenvalues.

REFERENCES

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