Abstracts of MMA2009, May 27 - 30, 2009, Daugavpils, Latvia

COMPARISON OF SPEEDS OF CONVERGENCE IN CERTAIN FAMILIES OF FUNCTIONAL SUMMABILITY METHODS

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Speeds of convergence in certain family of summability methods for functions are compared in the talk. The results introduced here extend the results proved in [1] for "matrix case" to "integral case" and are partly published in [2].

1. Let us denote by X the set of the functions $x = x(u)$ defined for $u \geq 0$, bounded and measurable by Lebesgue on every finite interval $[0, u_0]$. Suppose that A is a transformation of functions $x = x(u)$ (or, in particular, of sequences $x = (x_n)$) into functions $Ax = y = y(u) \in X$. If the limit $\lim_{u\to\infty} y(u) = s$ exists then we say that $x = x(u)$ is convergent to s with respect to the summability method A, and write $x(u) \to s(A)$.

One of the basic notions in our talk is the notion of speed of convergence. Let $\lambda = \lambda(u)$ be a positive function from X such that $\lambda(u) \to \infty$ as $u \to \infty$. We say that a function $x = x(u)$ is convergent to s with speed λ if the finite limit $\lim_{u\to\infty} \lambda(u)\,[x(u)-s]$ exists. In order to characterize the speed of convergence of x also the estimation $\lambda(u)\,[x(u) - s] = O(1)$ as $u \to \infty$, is used. We say that x is convergent with speed λ with respect to the summability method A if the function $Ax = y = y(u) \in X$ is convergent with speed λ.

2. We discuss a Riesz-type family $\{A_\alpha\}$ of summability methods A_α, where $\alpha > \alpha_0$ and α_0 is some fixed number, and which transform functions $x = x(u)$ into functions $A_\alpha x = y_\alpha(u)$. This family is defined with the help of relation $A_\beta = C_{\gamma,\beta} \circ A_\gamma \quad (\beta > \gamma > \alpha_0)$, where $C_{\gamma,\beta}$ is certain integral transformation (see e.g. [2]). For example, the Riesz methods (R, α) and certain generalized Nörlund methods $(N, p_\alpha(u), q(u))$ form Riesz-type families. A Riesz-type family has the monotony property: $x(u) \to s\,(A_\gamma) \Longrightarrow x(u) \to s\,(A_\beta)$ for any $\beta > \gamma > \alpha_0$.

It is important to be able to compare the speed of convergence of $x = x(u)$ with respect to different methods A_γ and A_β if $\beta > \gamma$ and choose a method A_α close to optimal for converging x. For a given speed $\lambda = \lambda(u)$ and a fixed number $\gamma > \alpha_0$ the speeds $\lambda_\beta = \lambda_\beta(u)$ and $\lambda_\delta = \lambda_\delta(u)$ $(\beta > \delta > \gamma)$ can be found (see [2]) such that for all $\beta > \delta > \gamma$ the following implications are true:

$$\lambda(u)\,[y_\gamma(u) - s] \to t \Longrightarrow \lambda_\beta(u)\,[y_\beta(u) - s] \to t,$$

$$\lambda(u)\,[y_\gamma(u) - s] = O(1) \Longrightarrow \lambda_\beta(u)\,[y_\beta(u) - s] = O(1),$$

$$\lambda(u)\,[y_\gamma(u) - s] = O(1),\ \lambda_\beta(u)\,[y_\beta(u) - s] \to t \Longrightarrow \lambda_\delta(u)\,[y_\delta(u) - s] \to t.$$

REFERENCES

[1] U. Stadtmüller and A. Tali. Comparison of certain summability methods by speeds of convergence, *Analysis Mathematica*, **29** (3), 2003, 227–242.

[2] A. Šeletski and A. Tali. Comparison of speeds of convergence in Riesz-type families of summability methods, *Proc. Estonian Acad. Sci.*, **57** (2), 2008, 70–34.