COMPARISON OF SPEEDS OF CONVERGENCE IN CERTAIN FAMILIES OF FUNCTIONAL SUMMABILITY METHODS

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Speeds of convergence in certain family of summability methods for functions are compared in the talk. The results introduced here extend the results proved in [1] for "matrix case" to "integral case" and are partly published in [2].

1. Let us denote by $X$ the set of the functions $x = x(u)$ defined for $u \geq 0$, bounded and measurable by Lebesgue on every finite interval $[0, u_0]$. Suppose that $A$ is a transformation of functions $x = x(u)$ (or, in particular, of sequences $x = (x_n)$) into functions $Ax = y = y(u) \in X$. If the limit $\lim_{u \to \infty} y(u) = s$ exists then we say that $x = x(u)$ is convergent to $s$ with respect to the summability method $A$, and write $x(u) \to s(A)$.

One of the basic notions in our talk is the notion of speed of convergence. Let $\lambda = \lambda(u)$ be a positive function from $X$ such that $\lambda(u) \to \infty$ as $u \to \infty$. We say that a function $x = x(u)$ is convergent to $s$ with speed $\lambda$ if the finite limit $\lim_{u \to \infty} \lambda(u) |x(u) - s|$ exists. In order to characterize the speed of convergence of $x$ also the estimation $\lambda(u) |x(u) - s| = O(1)$ as $u \to \infty$, is used. We say that $x$ is convergent with speed $\lambda$ with respect to the summability method $A$ if the function $Ax = y = y(u) \in X$ is convergent with speed $\lambda$.

2. We discuss a Riesz-type family $\{A_\alpha\}$ of summability methods $A_\alpha$, where $\alpha > \alpha_0$ and $\alpha_0$ is some fixed number, and which transform functions $x = x(u)$ into functions $A_\alpha x = y_{\alpha}(u)$. This family is defined with the help of relation $A_\beta = C_{\gamma, \beta} \circ A_\gamma$ ($\beta > \gamma > \alpha_0$), where $C_{\gamma, \beta}$ is certain integral transformation (see e.g. [2]). For example, the Riesz methods $(R, \alpha)$ and certain generalized Nörlund methods $(N, p_{\alpha}(u), q(u))$ form Riesz-type families. A Riesz-type family has the monotony property: $x(u) \to s(A_\gamma) \Rightarrow x(u) \to s(A_\beta)$ for any $\beta > \gamma > \alpha_0$.

It is important to be able to compare the speed of convergence of $x = x(u)$ with respect to different methods $A_\gamma$ and $A_\beta$ if $\beta > \gamma$ and choose a method $A_\alpha$ close to optimal for converging $x$. For a given speed $\lambda = \lambda(u)$ and a fixed number $\gamma > \alpha_0$ the speeds $\lambda_\beta = \lambda_\beta(u)$ and $\lambda_\delta = \lambda_\delta(u)$ ($\beta > \delta > \gamma$) can be found (see [2]) such that for all $\beta > \delta > \gamma$ the following implications are true:

$$
\lambda(u) |y_\gamma(u) - s| \to t \Rightarrow \lambda_\beta(u) |y_\beta(u) - s| \to t,
$$

$$
\lambda(u) |y_\gamma(u) - s| = O(1) \Rightarrow \lambda_\beta(u) |y_\beta(u) - s| = O(1),
$$

$$
\lambda(u) |y_\gamma(u) - s| = O(1), \ \lambda_\beta(u) |y_\beta(u) - s| \to t \Rightarrow \lambda_\delta(u) |y_\delta(u) - s| \to t.
$$

REFERENCES


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