

ON THE EIGENVALUE PROBLEMS FOR STATIONARY DIFFERENTIAL AND DIFFERENCE OPERATORS WITH COUPLED BOUNDARY CONDITIONS

SVAJŪNAS SAJAVIČIUS

Faculty of Social Informatics, Mykolas Romeris University

Ateities 20, LT-08303, Vilnius, Lithuania

E-mail: svajunas@mruni.eu

We consider the eigenvalue problem for one-dimensional differential operator with given nonlocal coupled boundary conditions,

$$\frac{d^2u}{dx^2} + \lambda u = 0, \quad 0 < x < 1, \quad (1)$$

$$u(0) = \gamma_0 u(1), \quad (2)$$

$$\frac{du}{dx} \Big|_{x=0} = \gamma_1 \frac{du}{dx} \Big|_{x=1}, \quad (3)$$

and its finite-difference counterpart,

$$\frac{U_{j-1} - 2U_j + U_{j+1}}{h^2} + \lambda U_j = 0, \quad j = 1, 2, \dots, N-1, \quad hN = 1, \quad (4)$$

$$U_0 = \gamma_0 U_N, \quad (5)$$

$$\frac{U_1 - U_0}{h} = \gamma_1 \frac{U_N - U_{N-1}}{h}, \quad (6)$$

where $\gamma_0, \gamma_1 \in \mathbb{R}$, $\gamma_0 + \gamma_1 \neq 0$. Such problem with $\gamma_0 = 0$ and $\gamma_1 = \beta$, $|\beta| > 1$, in finite-difference aspect was investigated by A. V. Gulín et al. (see [1] and references therein). We also briefly consider the similar two-dimensional differential and finite-difference problems.

The main aim of our work is to investigate the dependence of the qualitative structure of the spectrum of problems (1)–(3) and (4)–(6) on the parameters γ_0, γ_1 , i.e., to formulate conditions of existence of zero, positive, negative or complex eigenvalues. We use technique, which is used, for example, in papers [2; 3] to investigate similar problems with other types of nonlocal conditions.

REFERENCES

- [1] A. V. Gulín, N. I. Ionkin and V. A. Morozova. *The Stability of Nonlocal Difference Schemes*. URSS, Moscow, 2008. (In Russian)
- [2] R. Čiupaila, Ž. Jesevičiūtė and M. Sapagovas. On the Eigenvalue Problem for One-Dimensional Differential Operator with Nonlocal Integral Condition. *Nonlinear Analysis: Modelling and Control*, **9** (2), 2004, 109 – 116.
- [3] M. P. Sapagovas and A. D. Štikonas. On the Structure of the Spectrum of a Differential Operator with a Nonlocal Condition. *Differential Equations*, **41** (7), 2005, 961 – 969.