

ON INTEGRATION OVER FUZZY SET WITH RESPECT TO FUZZY VALUED MEASURE

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There are many works where measures of fuzzy sets and integrals over fuzzy sets were studied. A detailed survey of the most important concepts and results in this direction can be found in [2],[3]. As different from this approach our interest is to develop a theory of measure and integral where not only sets are fuzzy, but also measure and integral take fuzzy real values in the sense of [4].

The aim of the present talk is to define and investigate fuzzy valued integral $\int_E f d\mu$ of a function $f : X \rightarrow \mathbb{R}$ over a fuzzy set $E \in [0, 1]^X$ with respect to a fuzzy valued measure μ . In order to achieve this purpose we introduce the concept of a fuzzy valued measure (first defined in [1] and developed in our talk at the conference FSTA - "Fuzzy Sets: Theory and Applications" in 2008 in Slovakia). We assume that f is a measurable function with respect to a finite measure ν defined on a σ -algebra $\Phi \subset 2^X$ of crisp sets and then we extend measure ν to a fuzzy valued measure μ defined on a T -tribe $\Sigma \subset [0, 1]^X$ of fuzzy sets in the case when operations for fuzzy sets and fuzzy real numbers are based on a t -norm T [5]. By analogy with classical case we define an integral stepwise, first considering the case of characteristic functions, then extending it for simple non-negative measurable functions and finally for non-negative measurable functions. The basic properties of fuzzy valued integral are proved.

By using integral we define a fuzzy norm of functions on a fuzzy set E and consider functional space $\mathcal{L}(E)$. That gives us an opportunity to investigate on a fuzzy set E the error of approximation of $f \in \mathcal{L}(E)$.

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