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CONVEX EXTENSION OF FUNCTIONS AND APPLICATIONS IN MATHEMATICAL PROGRAMMING

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The convex extension f_c of function $f : X \to R(X \subset R^n)$ is the majorant convex function $f_c : convX \Rightarrow R$ where $f_c(x) = f(x)$ if $x \in X$. The function $f : X \to R$, $X \subset R^n$, is called discrete-convex if for all $x_i \in X(i = 1, \ldots, n+1); \lambda_i \geq 0(i = 1, \ldots, n+1)$ and $\sum_{i=1}^{n+1} \lambda_i = 1$; $\sum_{i=1}^{n+1} \lambda_i x_i \in X$ holds $f(\sum_{i=1}^{n+1} \lambda_i x_i) \leq \sum_{i=1}^{n+1} \lambda_i f(x_i)$. The use of all n+1 elements convex combinations follows from the well-known theorem of Caratheodory. Important special case is $X = Z^n$, where $Z^n = \underbrace{Z \times \ldots \times Z}_{n}$ and Z={. . . , -1,0,1, . . . } The graph of a discrete-convex function is a part of the graph of a convex function.

Theorem 1
The function $f : X \to R$, $(X \subset R^n)$ can be extended to the convex function on $convX$ if f is a discrete -convex on X. The convex extension f_c of f is

$$f_c(x) = \min_{x_i, \lambda_i} \Big\{ \sum_{i=1}^{n+1} \lambda_i f(x_i) \mid x = \sum_{i=1}^{n+1} \lambda_i x_i; \lambda_i \geq 0(i = 1, \ldots, n+1), x_i \in X(i = 1, \ldots, n+1) \Big\} \quad (1)$$

Theorem 2
The convex extension f_c of f is

$$f_c(x) = \begin{cases} \max\limits_{a,b} \{\langle a, x\rangle + b \mid \langle a, y\rangle + b \leq f(y), y \in X\}, x \notin X \\ f(x), x \in X \end{cases} \quad (2)$$

The convex extension is so called point-wise maximum over all linear functions not exceeding the given function. The convex function of a discrete-convex function is a piecewise linear function. Each discrete-convex function has a unique convex extension. The class of discrete-convex functions is the largest one to be extended to the convex functions. A class of iteration methods of local searching for solving discrete mathematical programming problems is developed. On each step of the iteration the calculation of the value of objective function is required only on some vertices of some kind of unit cube.