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ON NUMERICAL REALIZATION OF APOSTERIORI REGULARIZATION PARAMETER CHOICE RULES IN (ITERATED) TIKHONOV METHOD

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We consider a linear problem $Au = f$, $f \in \mathcal{R}(A)$, where $A \in \mathcal{L}(H, F)$ is a continuous operator between infinite-dimensional Hilbert spaces H and F. We do not suppose that range $\mathcal{R}(A)$ is closed. For approximation of the solution u_* of this ill-posed problem we use the Tikhonov method $u_\alpha = (\alpha I + A^*A)^{-1}A^*f_\delta$ or the iterated Tikhonov method $u_\alpha = u_{\alpha,m}$, where $u_{i,\alpha} = (\alpha I + A^*A)^{-1}(\alpha u_{i-1,\alpha} + A^*f_\delta)$ $(i = 1, 2, \ldots, m)$. Here $\alpha > 0$, I is the identity operator and f_δ is a known approximation to exact data f with noise level δ: $\|f_\delta - f\| \le \delta$. To choose the regularization parameter $\alpha = \alpha(\delta)$ we consider several quasioptimal parameter choice rules (see [1]) in the form $\varphi(\alpha) = b\delta$, $b = \text{const}$, where $\varphi(\alpha) = \langle Au_{\alpha,m} - f_\delta, Au_{\alpha,m+1} - f_\delta\rangle^{1/2}$ in case of the modified discrepancy principle, $\varphi(\alpha) = \alpha^{-k+1/2}\langle A^*(Au_{\alpha,m+k} - f_\delta), Au_{\alpha,m+k} - f_\delta\rangle$, $k \in \mathbb{N}$ in case of Rule R1, $\varphi(\alpha) = \|Au_{\alpha,m+1} - f_\delta\|^{-1}\langle Au_{\alpha,m} - f_\delta, Au_{\alpha,m+1} - f_\delta\rangle$ in case of the monotone error rule and $\varphi(\alpha) = \dfrac{\kappa_\alpha\|A^*(Au_{\alpha,m+1} - f_\delta)\|^2}{\sqrt{\alpha}\langle AA^*(Au_{\alpha,m+1} - f_\delta), Au_{\alpha,m+2} - f_\delta\rangle^{1/2}}$, $\kappa_\alpha = (1 + \alpha\|A\|^{-2})^{1/2}$ in case of Rule R2 [2]. These rules require computing of additionally iterated approximations. We propose for the realization of these parameter choice rules alternative numerical schemes, using instead of additional iterations linear combinations of approximations with different parameters (see [3]). We report also numerical experiments with test problems from P. C. Hansen's "Regularization toolbox".

REFERENCES

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