Abstracts of MMA2009, May 27 - 30, 2009, Daugavpils, Latvia

ASYMPTOTIC ANALYSIS OF LINEAR IMPULSE DYNAMICAL SYSTEM WITH MARKOV JUMPS

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Let $\{y(t), t \geq 0\}$ be the right continuous homogeneous Markov process on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with the infinitesimal operator $\mathcal{Q}$ defined by the formula

$$\mathcal{Q}v(y) = a(y) \int_Y (v(\xi) - v(y)) P(y, d\xi)$$

Y is a compact, $P(y, A)$ is a transition probability of a Markov chain with Feller's property. We assume that the spectrum of the operator $\mathcal{Q}$ has the form $\sigma(\mathcal{Q}) = \alpha\sigma_{-\rho} \cup \{0\}, \sigma_{-\rho} \subset \{z \in C : \mathbf{R}e \leq -\rho < 0\}$ and zero has the multiplicity one. The process $y(t)$ is ergodic and has the unique invariant probability measure $\mu \in C^*(Y)$. Let $\{\tau_j, j \in \mathbf{N}\}$ be switching moments of the above Markov process. Time periods between jumps are exponential distributed with intensity $a(y)$. We consider the system of

- the differential equation for the linear variable

$$\frac{dx(t)}{dt} = \varepsilon f_1(t, y(t), z(t))x(t) + \varepsilon^2 f_2(t, y(t), z(t))x(t), t \in (\tau_{j-1}, \tau_j), j \in \mathbf{N}$$

- the differential equation for the nonlinear variable

$$\frac{dz(t)}{dt} = \varepsilon h_1(t, y(t), z(t)), t \in [\tau_{j-1}, \tau_j), j \in \mathbf{N}$$

- and the jump equation for the linear variable

$$x(t) = x(t-) + \varepsilon g(t-, x(t-), y(t-), y(t), z(t)), t \in \{\tau_j, j \in \mathbf{N}\}$$

We have investigated the stability of this system with respect to the variable $x(t)$. After the transition to the fast time it was proved,that $\{x_\varepsilon(t), y_\varepsilon(t), z_\varepsilon(t)\}$ is the Markov process and derived its infinitesimal generator. Analyzing the infinitesimal operator of the above Markov process we construct a stochastic approximation in a form of the system of stochastic differential equations

$$\begin{aligned} dX &= B_{11}(Z)Xdt + B_{12}(Z)Xdw_{11}(t) + B_{13}(Z)Xdw_{12}(t) \\ dZ &= B_{21}(Z)dt + B_{22}(Z)dw_{21}(t) \end{aligned}$$

One can prove that processes $\{x_\varepsilon(t), z_\varepsilon(t)\}$ converge weakly as to the diffusion processes $\{X(t), Z(t)\}$. Under ergodic properties of $Z(t)$ one can calculate Lyapunov index for exponential stability analysis of $X(t)$. As the example the model of stochastic oscillator $\ddot{x} + \delta\dot{x} + (\omega^2 + h\cos(\nu t + y(t)) = 0$ was considered.