

AGGREGATION OPERATORS AND T-NORM BASED OPERATIONS WITH L-FUZZY REAL NUMBERS

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Our work deals with a fuzzy analogue of a real number. In the literature on fuzzy mathematics one can find several different schemes for defining fuzzy numbers. We consider the notion originating from B.Hutton paper [1] and later developed by several authors.

Let $L = (L, \wedge, \vee)$ be a completely distributive lattice with lower and upper bounds $0_L, 1_L \in L$. An L -fuzzy real number is a function $z: \mathbb{R} \rightarrow L$ such that

- (i) z is non-increasing;
- (ii) $\bigwedge_x z(x) = 0_L, \bigvee_x z(x) = 1_L$;
- (iii) z is left semi-continuous, i.e. $\bigwedge_{t < x} z(t) = z(x)$.

The set of all fuzzy real numbers is called the fuzzy real line and is denoted by $\mathbb{R}(L)$. The operations of L -fuzzy addition and L -fuzzy multiplication as they are defined in [2] are jointly continuous extensions of addition and multiplication from the real line \mathbb{R} to the L -fuzzy real line $\mathbb{R}(L)$.

The aim of this talk is to present alternative definitions for arithmetic operations with L -fuzzy numbers which are based on a triangular norm (recall that a triangular norm, or a t -norm for short, is an associative, commutative binary operation on a lattice L which is non-decreasing in each argument and has the neutral element 1_L [3]). For this aim we use the t -norm extension \tilde{A} of an aggregation operator A which is defined by the following formula [4]:

$$\tilde{A}(z_1, \dots, z_n)(x) = \bigvee_{x=A(x_1, \dots, x_n)} T(z_1(x_1), \dots, z_n(x_n)),$$

where $z_1, \dots, z_n \in \mathbb{R}(L)$, $x_1, \dots, x_n \in \mathbb{R}$ and T is a t -norm. Basic algebraic properties of these arithmetic operations are discussed. Examples illustrating the role of a t -norm in the definition of operations are given. In particular we consider the cases of minimum, product and Lukasiewicz t -norms.

REFERENCES

- [1] B. Hutton. Normality in Fuzzy Topological Spaces. *J.Math.Anal.Appl.*, **50** 1975, 74 – 79.
- [2] R. Lowen. On $(\mathbb{R}(L); \oplus)$. *Fuzzy Sets and Systems*, **10** 1983, 203 – 209.
- [3] E.P. Klement, R. Mesiar, E. Pap. *Triangular Norms*. Kluwer Academic Publishers, Dordrecht, 2000.
- [4] A. Takaci. General aggregation operators acting on fuzzy numbers induced by ordinary aggregation operators. *Novi Sad J. Math.*, **33** (2), 2003, 67 – 76.