Abstracts of MMA2009, May 27 - 30, 2009, Daugavpils, Latvia

NUMERICAL STUDY OF THE ROSENSWEIG INSTABILITY IN A MAGNETIC FLUID SUBJECT TO DIFFUSION OF MAGNETIC PARTICLES

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@page=1 Taking into account the process of diffusion of ferromagnetic particles under the action of a nonuniform magnetic field is a distinctive feature of a contemporary mathematical modeling of hydrostatics problems for a magnetic fluid. The steady-state distribution of the volumetric particle concentration C in the fluid volume V is described by the equation $\nabla \cdot (\nabla C - \xi C L(\xi H) \nabla H) = 0$ with the Robin-type boundary condition $\partial C/\partial n - \xi L(\xi H)(\partial H/\partial n)C = 0$ and the condition of particle mass conservation $\int_V C dV = C_0 V$ where L is the Langevin function; H, the magnetic-field intensity; $\xi = \mu_0 m/kT$; μ_0, the magnetic constant; m, the magnetic moment of a particle; k, the Boltzmann constant; T, the fluid temperature; C_0, a constant corresponding to a uniform distribution of particles. The exact solution of the problem is given in [1] and is of the form $C = \varphi C_0 V \Big/ \int_V \phi dV$, $\varphi = \sinh(\xi H)/(\xi H)$. The present study is devoted to the classical problem on stability of a horizontal semi-infinite layer of a magnetic fluid under the influence of gravity and a uniform magnetic field normal to the plane free surface [2]. It is well-known that development of small perturbations leads to the formation of a periodical peak-shaped structure on the surface when the magnetic-field intensity exceeds a critical value. The numerical modeling of the overcritical surface shapes is based on an axisymmetric statement. The coupled problem consists of three interconnected subproblems – on equilibrium shapes of a free surface, on a structure of magnetic field, on diffusion of magnetic particles. The generalized Young-Laplace parametric equations are governing equations in the first subproblem and the Maxwell equations formulated in terms of the potential of magnetic field, in the second. For the first the finite-difference scheme [3] and for the second, the finite-element method are used. The particle concentration C is computed by the formula above. Note that heretofore the problem has been solved in the uniform concentration approximation [4]. Numerical results show a significant influence of the particle diffusion on the overcritical shapes. Specifically, amplitude of peaks arising on the surface is appreciably higher and corresponds to experimental data better than in the case of the uniform concentration [4; 5].

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