Abstracts of MMA2009, May 27 - 30, 2009, Daugavpils, Latvia

ON FULLY DISCRETE COLLOCATION METHODS FOR SOLVING WEAKLY SINGULAR INTEGRO-DIFFERENTIAL EQUATIONS

RAUL KANGRO and ENN TAMME

Faculty of Mathematics and Computer Science, University of Tartu

Liivi 2, 50409, Tartu, Estonia

E-mail: `raul.kangro@ut.ee, enn.tamme@ut.ee`

A popular class of methods for solving initial or boundary value problems for weakly singular integro-differential equations of the form

$$u^{(n)}(t) = \sum_{i=0}^{n-1} a_i(t)u^{(i)}(t) + \sum_{i=0}^{n} \int_0^b K_i(t,s)u^{(i)}(s)ds + f(t)\,,\ 0 \le t \le b,$$

$$\sum_{j=0}^{n-1} \left[\alpha_{ij}u^{(j)}(0) + \beta_{ij}u^{(j)}(b)\right] = d_i\,, \quad i = 1,\dots,n\,,$$

is piecewise polynomial collocation method. In order to implement those methods one has to compute exactly certain integrals that determine the linear system to be solved. Unfortunately those integrals usually can not be computed exactly and even when analytic formulas exist, their straightforward application may cause unacceptable roundoff errors resulting in apparent instability of those methods in the case of highly nonuniform grids. Therefore it is very useful to consider fully discrete analogs of the collocation methods, where the system integrals are computed by using quadrature formulas.

We discretize the integrals of the form $\int_0^b K_i(t,s)u_m^{(i)}(s)\,ds$, where K_i are the kernels of the integral operators and u_m is the approximate solution corresponding to the collocation method, as follows. Namely, we introduce a nonuniform grid of the form

$$\Pi_1 = \{-b, -b\left(\tfrac{M_i-1}{M_i}\right)^{\rho_i}, \dots, -b\left(\tfrac{1}{M_i}\right)^{\rho_i}, 0, b\left(\tfrac{1}{M_i}\right)^{\rho_i}, \dots, b\left(\tfrac{M_i-1}{M_i}\right)^{\rho_i}, b\},$$

where $M_i > 0,\ \rho_i \ge 1$, and define a division of $[0,b]$ into subintervals by the points of the form $[0,b] \cap \{t-s\ :\ s \in \Pi_1\} \cup \Pi_0$, where Π_0 is the grid of the collocation method. In each of the subintervals we use the Gaussian quadrature formula with $n_i,\ 2n_i \ge m+n-i$ knots, where $m-1$ is the order of polynomials in the collocation method. We show that the additional error (in the L^∞ norm) caused by this discretization of i-th integral operator is bounded by the quantity

$$c\begin{cases} M_i^{-\rho_i(1-\nu_i)}, & 1 \le \rho_i < \frac{2n_i-m-n+i+1}{1-\nu_i}, \\ M_i^{-(2n_i-m-n+i+1)}(\log(M_i)+1), & \rho_i = \frac{2n_i-m-n+i+1}{1-\nu_i}, \\ M_i^{-(2n_i-m-n+i+1)}, & \rho_i > \frac{2n_i-m-n+i+1}{1-\nu_i}, \end{cases}$$

where ν_i is a parameter characterizing the singularity of the kernel K_i on the diagonal $t=s$, and give choices for ρ_i and M_i in the case of various integro-differential equations so that the order of convergence of the original collocation methods for those equations is preserved.