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FUZZY POS CATEGORY AND AGGREGATION OF FUZZY ORDER RELATIONS

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Since the first introduction of the concept of a fuzzy set by L. A. Zadeh and its generalization by J. A. Goguen [1] in the second half of the 20th century, fuzzy analogues of basic concepts of classical mathematics were introduced and investigated, in particularly fuzzy order relation [5]. In our work, we use the notion of a fuzzy order relation and construct a fuzzy POS category (L-POS) whose objects are fuzzy ordered sets and morphisms - "potential" order-preserving mappings (in the fuzzy sense); for the concept of fuzzy category see [4]. Actually, we involve L-fuzzy subclass of the class of morphisms as a mapping from the class of morphisms to GL-monoid L:

$$\mu : MOR(L\text{-}POS) \to L.$$

The intuitive meaning of the value $\mu(f)$ is the degree to which a morphism f is an order-preserving mapping.

We continue by constructing an aggregation process in the fuzzy POS category.
Let $n \geq 2$, $A : [0,1]^n \to [0,1]$ and let $R_1, R_2, ..., R_n$ be fuzzy relations ($R_i : X \times X \to L$).
An aggregation fuzzy relation R_A ($R_A : X \times X \to L$) is described by the formula

$$R_A(x,y) = A(R_1(x,y), ..., R_n(x,y)),\ x,y \in X.$$

We say an aggregation operator A preserves a given property of fuzzy relations if for all fuzzy relations $R_1, R_2, ..., R_n$ having this property, R_A also has this property. It is important to investigate which aggregation operators are able to preserve properties of fuzzy relations in aggregation process. There are many works devoted to this topic, see e.g. [3]. In our work we involve the degree to which aggregation operator preserves properties of fuzzy relations. In order to calculate how good the operator A preserves the properties of fuzzy relations $R_1, R_2, ..., R_n$ we use the mapping ξ: $\xi(A) = \inf\limits_i \mu_i$, where μ_i is the degree to which an aggregation operator A preserves the property of the corresponding fuzzy relation R_i. Further we investigate properties and behavior of the mapping ξ and illustrate the obtained theoretical results by the real world example.

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