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EFFICIENT MULTIGRID METHOD ON HIERARCHICAL TRIANGULAR GRIDS FOR THE BIOT'S CONSOLIDATION PROBLEM

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The classical quasi-static Biot model for soil consolidation, describes the time dependent interaction between the deformation of an elastic porous material and the fluid flow inside of it. This model can be formulated as a system of partial differential equations for the unknowns displacement $\mathbf{u} = (u, v)$ and pressure p. Here, we consider the case of a homogeneous, isotropic and incompressible medium Ω so the governing equations are given by

$$\begin{aligned} &-\mu\Delta\mathbf{u} - (\lambda+\mu)\operatorname{grad}\operatorname{div}\mathbf{u} + \operatorname{grad} p = \mathbf{g}(\mathbf{x}, t), \\ &\frac{\partial}{\partial t}(\operatorname{div}\mathbf{u}) - \frac{\kappa}{\eta}\Delta p = f(\mathbf{x}, t), \quad \mathbf{x} \in \Omega,\ 0 < t \leq T. \end{aligned}$$

where λ and μ are the Lamé coefficients, κ is the permeability of the porous medium and η the viscosity of the fluid. A stabilized finite element scheme for the poroelasticity equations, based on the perturbation of the flow equation, was proposed in [1]. This allowed us using continuous piecewise linear approximation spaces for both displacements and pressure, obtaining solutions without oscillations independently of the chosen discretization parameters.

To approximate solutions of mathematical physics problems defined on irregular domains it is very common to apply regular refinement to an unstructured input grid. Assuming that the coarsest grid is rough enough in order to fit the geometry of the domain, a hierarchy of globally unstructured grids is generated. This kind of meshes are suitable for use with geometric multigrid and this method is implemented using stencil-based operations to remove the limitations on the size of the problem that can be solved by using this process of refinement. In this framework, we are interested in the design of an efficient geometric multigrid method on hierarchical triangular grids for a linear finite element discretization of the poroelasticity problem. To design this geometric multigrid method, a Local Fourier Analysis on triangular grids, which was introduced in [2], is necessary. This tool permits to choose the suitable components of the algorithm in order to obtain an efficient multigrid method.

REFERENCES

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