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DISCRETIZATION OF ELLIPTIC CONTROL PROBLEMS BY FINITE ELEMENTS

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We consider an optimal control problem for systems by elliptic partial differential equations, with state constraints and a *minimax* objective function. Using the finite element method theory, we discretize the problem and linearize the minimax control problem.
Let Ω be a bounded polygonal domain in $\mathbb{R}^k$ with a piecewise smooth boundary $\Gamma = \Gamma_1 \bigcup \Gamma_2 \bigcup \Gamma_3$.The state constrained boundary control problem is stated as

$$\begin{array}{lllll} \min\limits_{u} & \max\limits_{\Omega} & y(x), & & \\ s.t. & -\nabla \cdot (K\nabla y) + by & = & f & in\ \Omega, \\ & y & \geq & \psi & in\ \bar{\Omega}, \\ & y & = & u & on\ \Gamma_1, \\ & y & = & g & on\ \Gamma_2, \\ & \frac{\partial y}{\partial \nu} & = & q & on\ \Gamma_3, \end{array} \tag{1}$$

where f, g and q are given functions and u represents the boundary control on Γ_1. We discrete the problem (1) by finite element method.

Theorem 1. *When k=2, let $\mathcal{T}_h$ be a valid triangulation of Ω. The discrete problem satisfies the discrete maximum principle for h small enough if there exists $\varepsilon > 0$ such that for all h, all the angles of the triangles of $\mathcal{T}_h$ are less than or equal to $\frac{1}{2}\pi - \varepsilon$.*

When the discrete maximum principle holds, a valid linear formulation of the discrete boundary control problem is

$$\begin{array}{ll} \min & s, \\ s.t. & \tilde{e}s - \tilde{\beta} \geq 0, \\ & A\beta + \tilde{A}\tilde{\beta} = F, \\ & \beta \geq \Psi, \\ & \tilde{\beta} \geq \tilde{\Psi}. \end{array} \tag{2}$$

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