

# DESIGN AND ANALYSIS OF ODE MODELS WITH VARIABLE TIME DELAYS FOR TUMOUR DEVELOPMENT

PETERIS DAUGULIS

*Daugavpils University*

Parades 1, LV-5400, Daugavpils, Latvia

E-mail: peteris.daugulis@du.lv

In this talk we present the main considerations related to design and basic stability and periodicity analysis of ODE and time delay ODE models for dynamics of systems consisting of solid tumours and their protein and vascular environments.

In [1] several ODE and time delay ODE systems have been defined encoding the most essential observations and assumptions about the development of systems enabling tumour development. An example of a model from this series is

$$\begin{cases} \dot{N} = f_1(E_{\tau_1})N \\ \dot{P} = f_2(E)N - \delta P \\ \dot{E} = f_3(P_{\tau_2})E - f_1(E_{\tau_1})E. \end{cases} \quad (1)$$

In order to uncover the basic features of the models such as feedback loops the Hopf point analysis of the models has to be performed. The main conclusion of [1] is the existence of Hopf bifurcations with nonzero time delays. This conclusion may lead to practical results since Hopf bifurcation imply existence of small periodic oscillations around equilibrium states. Such periodic oscillations may be enforced by suitable therapies and/or surgeries.

The models given in [1] can be generalized allowing time delays to be dependent on state variables. We describe such models and perform analysis similar to that given in [1].

## REFERENCES

- [1] Z.Agur, L.Arakelyan, P.Daugulis and Y.Ginosar. Hopf point analysis for angiogenesis models. *Discrete And Continuous Dynamical Systems - Series B*, **4** (1), 2004.