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ASYMPTOTICAL METHODS FOR CONDITIONAL SECOND MOMENTS OF LINEAR MARKOV DIFFERENCE EQUATIONS

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The paper deals with n-dimension linear stochastic difference equation in $\mathbb{R}^n$ defined by equality

$$x_t = A(y_t)\, x_{t-1},\ t \in \mathbb{N} \tag{1}$$

where $\{y_t, t \in \mathbb{N}\}$ is a homogeneous ergodic Feller Markov chain with invariant measure $\mu(dy)$ and transition probability $p(y, dz)$ on the metric compact space $\mathbb{Y}$, and $\{A(y), y \in \mathbb{Y}\}$ is uniformly bounded continuous $n \times n$ matrix function. We will suppose that matrix $A(y)$ has a near to constant form

$$A(y, \varepsilon) := M + \sum_{k=1}^{l} \varepsilon^k A_k(y), \tag{2}$$

where ε is a small positive parameter and matrix M has spectrum in a following form: $\sigma(M) = \sigma_0(M) \cup \sigma_\gamma(M)$ divided into two parts $\sigma_0(M) \subset \{|\lambda| = 1\}$ and $\sigma_\gamma(M) \subset \{|\lambda| \le \gamma < 1\}$. This problem arises in financial econometrics at the stock dynamical analysis within infinite time interval (see, for example, [2]). The paper proposes a convenient for application asymptotic method of stochastic stability analysis for the equation (1) with near to constant coefficients (2). Applying the methods and results of [1] we produce an algorithm which allows to reduce the above problem to testing of positive definition property of a solution of the specially constructed matrix equation or on spectral properties of linear continuous operator $(\mathbb{A}(\varepsilon)q)(y,\varepsilon) := \int\limits_Y A^T(z,\varepsilon)q(z)A(z,\varepsilon)p(y,dz)$ acting in the Banach space $\mathbf{V}$ of symmetric uniformly bounded continuous $n \times n$ matrix functions $\{q(y), y \in Y\}$ with norm $||q|| := \sup\limits_{y \in Y, |x|=1} |(q(y)x, x)|$. Our results are based on existence [1] such positive numbers d and ε_0 that for any $\varepsilon \in (0, \varepsilon_0)$ equation (1) with matrix (2) is exponentially mean square stable if and only if the equation $(\mathbb{A}(\varepsilon)q)(y,\varepsilon) - q(y,\varepsilon) = -I$ has solution in a form of Laurent series $q(y,\varepsilon) = \sum\limits_{k=-d}^{\infty} \varepsilon^k q_k(y)$, $d \ge 1$ with positive defined main part $\widehat{q}(y,\varepsilon) := \sum\limits_{k=-d}^{0} \varepsilon^k q_k(y)$.

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