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# ASYMPTOTIC METHODS FOR MARKOV DYNAMICAL SYSTEMS

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Averaging principle and diffusion approximation procedures are the most frequently used asymptotic methods for analysis of nonlinear dynamical systems subjected to permanent random perturbations (see, for example, [1] and references there). To employ these approaches to contemporary applied research of statistical nonlinear dynamics one should introduce a small positive parameter $\varepsilon \in (0, \varepsilon_0)$ and divide phase variables to slow $x^\varepsilon(t) \in \mathbf{X}$ with vector field $F(x^\varepsilon(t), y^\varepsilon(t), \xi^\varepsilon(t))$ and fast $y^\varepsilon(t) \in \mathbf{Y}$, which vector field is proportional to negative powers of $\varepsilon$, that is has a form $\varepsilon^{-1}G(y^\varepsilon(t)) + H(x^\varepsilon(t), y^\varepsilon(t), \xi^\varepsilon(t))$, where $\xi^\varepsilon(t)$ is fast oscillating random perturbations. We assume that $\xi^\varepsilon(t)$ is ergodic homogeneous Markov process with invariant measure $\mu(d\xi)$ on compact space $\mathbf{\Xi}$ defined by weak infinitesimal operator $\varepsilon^{-1}\mathbb{Q}$. According to [2] this mathematical model is called Markov dynamical systems defined as Markov process on $\mathbf{X} \times \mathbf{Y} \times \mathbf{\Xi}$ with weak infinitesimal operator $\mathbb{L}(\varepsilon)v(x,y,\xi) := (F(x,y,\xi), \nabla_x)v(x,y,\xi) + (H(x,y,\xi), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}(G(y), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}\mathbb{Q}v(x,y,\xi)$. The classical averaging principle suggests a good approximation $\bar{x}(t)$ of the slow motion $x^\varepsilon(t)$ on any finite time intervals can be obtained as a solution of averaged equation $\frac{d\bar{x}}{dt} = \bar{F}(\bar{x})$ where $\bar{F}(x) := \lim_{t\to\infty} \frac{1}{t}\int_0^t \int_{\mathbf{\Xi}} F(x, \bar{y}(s), \xi)\mu(d\xi)ds$, and $\bar{y}(s)$ is solution of "truncated" equation $\frac{d\bar{y}}{dt} = G(\bar{y})$. If $\bar{F}(x) \equiv 0$ and above mentioned "truncated" equation for $y(t)$ has unique asymptotically stable bounded solution one can ([2], [3], [4]) proceed to "slow" time $\varepsilon t$ and to construct stochastic approximation of initial dynamical systems in a form of stochastic Ito differential equation. One can prove that above averaged and diffusion approximation equations may be successfully used not only for approximate analysis of initial system on finite time interval but also for Lyapunov stability analysis of motion $x^\varepsilon(t)$ [4]. The paper also proposes method and algorithm of stochastic stability analysis for more often application-oriented dynamical systems [1] with unbounded solutions of "truncated" equation. Our approach may be of use also to approximating stochastic modeling of commonly encountered in financial econometrics impulse type Markov dynamical systems defined by weak infinitesimal operator $L(\varepsilon)v(x,y,\xi) := (F(x,y,\xi), \nabla_x)v(x,y,\xi) + (H(x,y,\xi), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}(G(y), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}a(\xi)\int_{\mathbf{\Xi}}(v(x+\varepsilon f(x,y,\xi), y+\varepsilon h(x,y,\xi), \zeta) - v(x,y,\xi))p(\xi, d\zeta)$ where $p(\xi, d\zeta)$ is transition probability of embedded in the compound Poisson process $\xi^\varepsilon(t)$ Markov chain.

## REFERENCES

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