Abstracts of MMA2009, May 27 - 30, 2009, Daugavpils, Latvia

ON STOCHASTIC VOLATILITY MODELING

JEVGEŅIJS CARKOVS & AIGARS EGLE

Riga Technical University

Kaļķu 1, LV-1628, Rīga, Latvia

E-mail: carkovs@latnet.lv; Aigars.Egle@parex.lv

Many econometrical studies (see [1],[2] and references there) have documented that financial time series tend to be highly heteroskedastic. This has many implications for many areas of macroeconomics and finance, including the term structure of interest rates, option pricing and dynamic capital-asset pricing theory. In the same time econometricians have also been very active in developing models of conditional heteroskedasticity. The most widely used models of dynamic conditional variance have been the ARCH models first introduced by [1]. In most general form, a univariate ARCH model makes conditional variance at time t a function of exogenous and lagged endogenous variables, time, parameters and past residuals. According to Nelson [2] approach to stochastic modeling one should fit to sampling data with in discrete time GARCH(1,1) process $\sigma^2_{t+1} = \omega_h + \sigma^2_t[\beta_h + h^{-1}\alpha_h {}_h Z^2_t]$ for conditional variance $\sigma^2_t := \mathbf{D}\left\{ \frac{S_{t+1}-S_t}{S_t} \Big/ {}_h Z_s, s \le t \right\}$ where h is small positive parameter, and $\{{}_h Z_t, t \in \mathbb{Z}\}$ are independent $\mathbf{N}(0, h)$ random variables. Under assumptions $1 - \alpha_h - \beta_h = h\theta + o(h), \omega_h = h\omega + o(h), \alpha_h = \frac{\sqrt{h}}{\sqrt{2}}\alpha + o(h)$ author of paper [2] derives continuous time approximation for conditional variance in a form of stochastic Ito differential equation

$$d\sigma^2_t = (\omega - \theta\sigma^2_t)dt + \alpha\sigma^2_t dw(t) \tag{1}$$

But sometimes as we will show analyzing real data, hypothesis on non-correlatedness of cumulative excess returns residuals may be rejected. Our paper discusses a possible correlation effect assuming that random process ${}_h Z_t$ as before is stationary $\mathbf{N}(0, h)$ but with correlation coefficient ρ. Applying the method and results of the paper [3] we have derived continuous time approximation for conditional variance in a form of diffusion process satisfying stochastic Ito differential equation

$$d\sigma^2_t = \left(\omega + \left(\frac{\alpha^2\rho^2}{1-\rho^2} - \theta\right)\sigma^2_t\right)dt + \alpha\sqrt{\frac{1+\rho^2}{1-\rho^2}}\sigma^2_t dw(t) \tag{2}$$

with coefficients dependent on correlation parameter ρ. Analyzing ergodic property of this equation we have shown that it is important to take into account possible serial correlation in conditional variance process. Our results are also illustrated and confirmed by statistic simulation.

REFERENCES

[1] R.F. Engle, T. Borslev. Modelling the Persistence of Conditional Variances. *Economic Reviews*, 5, 1986, 1 – 50.

[2] D. Nelson. ARCH Models as Diffusion Approximation. *Jurn. of Econometrics*, 45, 1990, pp. 7–38 .

[3] J. Carkovs. On Diffusion Approximation of Discrete Markov Dynamical Systems. In: *Proc. of World Congress of Science, Engineering and Technology, Paris, France, July 04–08, 2008*, Procidings of World Academy of Science, Engineering and Technology, v. 30, C. Ardil (Eds.), WASET, Paris, France, 2008, 1–6.