

ON USING SPLINES FOR THE APPROXIMATION OF A JOINT DENSITY FUNCTION

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This paper deals with the problem of the approximation of a density function underlying a given histogram on the rectangular grid $\Delta_x \times \Delta_y$, where $\Delta_x = \{x_0, x_1, \dots, x_n\}$ and $\Delta_y = \{y_0, y_1, \dots, y_m\}$ are two sequences of strictly increasing knots with $h_i^x = x_i - x_{i-1}$, $h_j^y = y_j - y_{j-1}$. Let

$$F_{x \times y} = \{f_{ij}, i = 1, \dots, n, j = 1, \dots, m\}$$

be a given histogram. The quantity f_{ij} presents the frequency of a data set on the subrectangle

$$R_{ij} = [x_{i-1}, x_i] \times [y_{j-1}, y_j].$$

We are interested in having a function $g(x, y)$ that satisfies the volume matching histopolation conditions

$$\iint_{R_{ij}} g(x, y) dx dy = h_i^x h_j^y f_{ij}, \quad i = 1, \dots, n, j = 1, \dots, m$$

The problem of histopolation is solvable, but not uniquely.

Different approaches to the solution of this problem by using splines are observed in this paper: the variational approach (e.g. [2]) and the approach, which concerns reproducing the main geometric characteristics of the histogram (positivity, directional monotonicity, local maxima and minima) by choosing the corresponding class of splines and creating free parameters which should be suitably chosen (e.g. [1]). One of the method for the solution of the problem of histopolation is considered in a more detailed way.

The possibility for the approximation of the joint distribution function by using splines is discussed.

REFERENCES

- [1] P. Costantini, F. Pelosi. Shape preserving histogram approximation . *Advances in Computational Mathematics* , **26** (1-3), 2007, 205 – 230 .
- [2] N. Dyn, W. H. Wong. On the characterization of non-negative volume-matching surface splines . *J. Approx. Theory*, **51** (1), 1987 , 1–10 .