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CONSERVATIVE DIFFERENCE SCHEMES ON ADAPTIVE GRIDS FOR WAVE EQUATION

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To computational methods for solving problems of mathematical physics the requirements of monotonicity, conservativeness and effectiveness are usually produced. Finite-difference methods satisfying these requirements are widely studied on uniform spatial-time grids and non-uniform spatial grids (see, for example, [1; 2]. The case of non-uniform time grid is not enough investigated by now. Particular results were received under assumption of Lipschits-continuity of operators of the scheme which leads to unnatural condition of quasi-uniformity of time grid. In [4] new a priori estimates of stability of three-level schemes with respect to the initial data on non-uniform grids in time were found. For various real problems it is more desirable to construct difference schemes in such a way that the grid analogs of conservation laws hold true for them. Schemes for which grid laws of energy conservation are satisfied are called conservative. Moreover the order of convergence of conservative difference schemes can be higher than those of non-conservative methods for the same problem in the case of coarse grids. What concerns effectiveness, the usage of adaptive grids considerably decreases the machine time of problem computing. In works [3; 4] a three-level scheme on non-uniform spatial-time grid written in non-conservative form was introduced.

In the presentation a new conservative three-level difference scheme of the second order of local approximation is considered for one-dimensional wave equation on non-uniform spatial-time grid:

$$y_{\bar{t}\hat{t}} + \left(\frac{h^2}{6} y_{\bar{t}\hat{t}\bar{x}}\right)_{\hat{x}} + Ay^{(\sigma_1,\sigma_2)} = \varphi, \quad (x,t) \in \hat{\omega}_{h\tau}, \quad y_0 = u_0, \quad y_1 = u_1,$$

$$\hat{\bar{\omega}}_{h\tau} = \{x_i = x_{i-1} + h_i, i = \overline{1, N-1}, x_0 = 0, x_N = l, t_n = t_{n-1} + \tau_n, n = \overline{1, N_0 - 1}, t_0 = 0, t_{N_0} = T\},$$

$$\sigma_1 = (2\tau_{n+1} + \tau_n)/(6(\tau_{n+1} + \tau_n)), \quad \sigma_2 = (\tau_{n+1} + 2\tau_n)/(6(\tau_{n+1} + \tau_n)).$$

For the introduced class of schemes with weights a priori estimates of stability and convergence with respect to the initial data and the right-hand side are received without requirement of Lipschitz-continuity of operators and quasi-uniformity of time grid. Theoretical results are confirmed by computational experiments.

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