ON SOLUTIONS OF THE EMDEN-FOWLER TYPE EQUATION

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We consider the boundary value problem (BVP)

\[ x'' = -q(t) |x|^p \text{sign } x, \quad (1) \]
\[ x(0) = x(1) = 0, \quad (2) \]

where \( q \in C([0, 1], R), \quad p > 0, \quad p \neq 1. \)

Our aim is to obtain conditions for existence of multiple solutions. We investigate the problem (1), (2) by reducing it to multiple quasi-linear problems of different types. Suppose that equation (1) can be written in the equivalent quasi-linear form

\[ x'' + k^2 x = F(t, x, x'). \quad (3) \]

Definition 1. We say that a solution \( \xi(t) \) of the problem (1), (2) is an \( i \)-type solution if for small enough \( \alpha > 0 \) the difference \( u(t; \alpha) = x(t; \alpha) - \xi(t) \) has exactly \( i \) zeros in the interval \((0, 1)\) and \( u(1; \alpha) \neq 0 \), where \( x(t; \alpha) \) is a solution of (3), which satisfies the initial conditions

\[ x(0; \alpha) = \xi(0) = 0, \quad x'(0; \alpha) = \xi'(0) + \alpha. \quad (4) \]

Theorem 2. Suppose that \( 0 < q_1 \leq q(t) \leq q_2 \) \( \forall t \in [0, 1] \). Then if there exists \( k \in (i\pi, (i+1)\pi), \quad i = 0, 1, 2, \ldots \), which satisfies the inequality

\[ \frac{k}{|\sin k|} < \beta \cdot \frac{p^{p-1}}{|p-1|} \left( \frac{q_1}{q_2} \right)^{\frac{1}{p-1}}, \quad (5) \]

where \( \beta \) is a positive solution of the equation \( \beta^p = \beta + (p-1) \cdot p^{\frac{p-1}{p}}, \) then there exists an \( i \)-type solution of the problem (1), (2).

Corollary 3. If there exist \( k_j \in (ij\pi, (ij+1)\pi), \quad j = 1, 2, \ldots, n \), which satisfy the inequality (5), then there exist at least \( n \) solutions of different types to the problem (1), (2).

REFERENCES