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ON SOLUTIONS OF THE EMDEN-FOWLER TYPE EQUATION

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We consider the boundary value problem (BVP)

$$x'' = -q(t)\,|x|^p \operatorname{sign} x, \tag{1}$$

$$x(0) = x(1) = 0, \tag{2}$$

where $q \in C\big([0,\,1], R\big), \quad p > 0, \quad p \neq 1.$

Our aim is to obtain conditions for existence of multiple solutions. We investigate the problem (1), (2) by reducing it to multiple quasi-linear problems of different types. Suppose that equation (1) can be written in the equivalent quasi-linear form

$$x'' + k^2\,x = F(t,\,x,\,x'). \tag{3}$$

DEFINITION 1. We say that a solution $\xi(t)$ of the problem (1), (2) is an i-type solution if for small enough $\alpha > 0$ the difference $u(t;\alpha) = x(t;\alpha) - \xi(t)$ has exactly i zeros in the interval $(0,1)$ and $u(1;\alpha) \neq 0$, where $x(t;\alpha)$ is a solution of (3), which satisfies the initial conditions

$$x(0;\alpha) = \xi(0) = 0, \qquad x'(0;\alpha) = \xi'(0) + \alpha. \tag{4}$$

THEOREM 2. *Suppose that* $\quad 0 < q_1 \leq q(t) \leq q_2 \quad \forall\, t \in [0,1]$. *Then if there exists* $k \in \big(i\pi, (i+1)\pi\big)$, $i = 0, 1, 2, \ldots$, *which satisfies the inequality*

$$\frac{k}{|\sin k|} < \beta \cdot \frac{p^{\frac{p}{p-1}}}{|p-1|} \cdot \left(\frac{q_1}{q_2}\right)^{\frac{1}{|p-1|}}, \tag{5}$$

where β *is a positive solution of the equation* $\quad \beta^p = \beta + (p-1)\cdot p^{\frac{p}{1-p}}, \quad$ *then there exists an* i*-type solution of the problem* (1), (2).

Corollary 3. If there exist $\quad k_j \in \big(i_j\pi, (i_j+1)\pi\big), \quad j = 1, 2, \ldots, n \quad$, which satisfy the inequality (5), then there exist at least n solutions of different types to the problem (1), (2).

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