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FAST SOLVERS OF INTEGRAL EQUATIONS OF THE SECOND KIND

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Consider the integral equation

$$u(x) = \int_0^1 K(x,y)u(y)\,dy + f(x), \quad 0 \le x \le 1, \tag{1}$$

where $f(x)$ is m-smooth and $K(x,y)$ is at least $2m$-smooth. Assume that the homogenous integral equation corresponding to (1) has in $L^2(0,1)$ only the trivial solution. Our aim is to design methods that produce approximate solutions u_n, $n \in \mathbb{N}$, such that

- given the values of f and K at $O(n)$ suitably chosen points, the parameters of u_n are available at the cost of $O(n)$ flops, and the accuracy

$$\sup_{0\le x\le 1} \mid u(x) - u_n(x) \mid \le cn^{-m} \max_{0\le j\le m} \sup_{0\le x\le 1} \mid f^{(j)}(x) \mid \tag{2}$$

  is acieved, where u is the solution of (1) and c is a constant that is independent of n and f;
- having determined the parameters of u_n, the value of u_n at any particular point $x \in [0,1]$ is available with the same accuracy as (2) at the cost of $O(1)$ flops.

We call such a method fast (C, C^m) solver of equation (1). In the talk, main ideas of the construction of fast solvers will be explained. Fast solvers are optimal methods in different senses, see [3]. In [3], a fast (C, C^m) solver of equation (1) was constructed on the basis of the piecewise polynomial Galerkin method combined with the two grid iterations to solve the Galerkin system. We construct [1] fast (C, C^m) solvers of equation (1) on the basis of quadrature methods, also combined with the two grid iterations to solve the corresponding system of linear algebraic equations. Similar fast solvers can be constructed on the basis of piecewise polynomial collocation method. In the case of integral equations on the real line, with the polynomial or exponential decay of $f(x)$ and $K(x,y)$, we construct [2] fast (C, C^m) solvers on the basis of wavelet Galerkin method with the Sloan improvement of the Galerkin solution combined with GMRES to solve the Galerkin system.

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