Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

STABILITY OF THE SPLINE COLLOCATION METHOD FOR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

MARE TARANG

Institute of Applied Mathematics, University of Tartu
Liivi 2, 50409, Tartu, Estonia
E-mail: mare.tarang@ut.ee

We consider the 1st order Volterra integro-differential equation (VIDE)

$$y'(t) = f(t, y(t) + \int_0^t K(t, s, y(s))\, ds, \quad t \in [0, T] \tag{1}$$

with the initial condition $y(0) = y_0$. In addition, we consider the 2nd order VIDE

$$y''(t) = f(t, y(t), y'(t)) + \int_0^t \mathcal{K}(t, s, y(s), y'(s))\, ds, \quad t \in [0, T], \tag{2}$$

with the initial conditions $y(0) = y_0$ and $y'(0) = y_1$. In order to solve (1) or (2), we use a step-by-step spline collocation method, where the polynomial splines may have arbitrary degree and defect. In addition, for (1) we investigate the case with one multiple node on each grid interval. A numerical stability of the method is defined as boundedness of the approximate solutions for the certain test equation on uniform grid. Necessary and sufficient conditions for the numerical stability of the same method for Volterra integral equations are found in [1]. These are completely determined by the eigenvalues of the transfer matrix from an interval to the next one. For VIDE stability conditions are found in [2; 3; 4]. We show that 1) for VIDE the numerical stability is also determined by the eigenvalues of the transfer matrix; 2) the eigenvalues of the transfer matrix for 1st order VIDE have the same geometrical multiplicity as the eigenvalues of the transfer matrix for integral equations; 3) in case of piecewise polynomial collocation method the numerical stability also depends on the parameters of certain test equation; 4) the eigenvalues of the transfer matrix for 2nd order VIDE have the same geometrical multiplicity as the eigenvalues of the transfer matrix for 1st order VIDE.

REFERENCES

[1] P. Oja. Stability of the spline collocation method for Volterra integral equations. *J. Integral Equations Appl*, **13** (2), 2001.

[2] P. Oja and M. Tarang. Stability of piecewise polynomial collocation for Volterra integro-differential equations. *Math. Model. Anal.*, **6** (2), 2001, 310 - 320.

[3] P. Oja and M. Tarang. Stability of spline collocation method for Volterra integro-differential equations. *Acta et Commentationes Universitatis Tartuensis de Mathematica*, **6** 2002, 37 - 49.

[4] M. Tarang. Stability of spline collocation method for second order Volterra integro-differential equations. *Math. Model. Anal.*, **9** (1), 2004, 79 - 90.