GENERALIZED RIEZ METHOD AND CONVERGENCE ACCELERATION

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In many fields of numerical mathematics we have some problems of convergence acceleration. Though mainly are used non-linear methods (see [3], in several cases are preferred linear methods. In this talk we deal with the possibilities to use generalized Riesz method for acceleration.

Let $X$ and $Y$ be Banach spaces and $L(X, Y)$ be a space of linear bounded operators from $X$ into $Y$. A sequence $x = (\xi_k) (\xi_k \in X)$ is called $\lambda$-bounded ($\lambda$-convergent) if $\beta_k = O(1)$ ($\exists \lim \beta_k$), while $\beta_k = \lambda_k (\xi_k - \xi)$ with $\xi = \lim \xi_k, \lambda = (\lambda_k)$ and $0 < \lambda_k \not\to$. Let $m_X^\lambda (c_X^\lambda)$ be a set of all $\lambda$-bounded ($\lambda$-convergent) sequences. A sequence $x = (\xi_k)$ is called summable by a generalized method $A = (A_{nk})$ if $y = (\eta_n)$ with $\eta_n = \sum_k A_{nk}\xi_k$ and $A_{nk} \in L(X, Y)$ is convergent. The transformation $A$ is called accelerating $\lambda$-boundedness ($\lambda$-convergence) if $Am_X^\lambda \subset m_Y^\mu$ with $\lim \mu_n/\lambda_n = \infty$. Let us denote by $(\Re, P_n)$ or shortly by $\Re$ the generalized Riesz method, defined by

$$R_{nk} = \begin{cases} R_n P_k & k = 0, 1, \ldots, n, \\ \theta & k > n, \end{cases}$$

where $P_k, R_n \in L(X, X)$, while $R_n$ is determined by

$$R_n \sum_{k=0}^n P_k \zeta = \zeta \quad (\zeta \in X, n \in \mathbb{N}_0).$$

In [2] are proved sufficient conditions for the inclusion $\Re m_X^\lambda \subset m_Y^\mu$. In [1] are studied several inverse theorems, so-called Tauberian theorems for generalized Riesz method $\Re$ in the case $\lambda_n = O(1)$ or $\mu_n = O(1)$. We study the case $\lambda_n \not\neq O(1)$ or $\mu_n \not\neq O(1)$ and prove so-called Tauberian remainder theorems for the method $\Re$.

REFERENCES

