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MATHEMATICAL MODELLING OF WET PRESSING IN PAPER MACHINE

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We consider a flow in highly compressible porous media (paper-felt system) which is compressed between two rotating rolls or between a rotating roll and a fixed plate. It is assumed that the porous layer consists of three phases. The flow of those three components is governed by the mass continuity equations and the generalized Darcy's laws[2; 3]: $\nabla(\varphi_f \boldsymbol{v}_f) = 0$; $\varphi_f(\boldsymbol{v}_f - \boldsymbol{v}_s) = -\frac{K_f}{\mu_f}\nabla p_f$, where index $f = w, a$. Two fluid phases indexed "w"(water) and "a"(gas), a solid phases indexed "s". Here $s = \frac{\varphi_w}{\varphi}$ is a saturation and $\varphi = 1 - \varphi_s$ is the porosity. The coefficient K_f depends on φ and s: $K_f(\varphi, s) = k\frac{\varphi}{(1-\varphi)^2}s^m$[2] or $K_f(\varphi, s) = k\frac{\varphi}{(1-\varphi^2)}s^m$[3] (the Kozeny-Carman equation).

In the first model [2] the condition $v_w^x = v_a^x = v_s^x = c$ is assumed to be valid, where c is constant machine speed. We get that the total pressure p_T depends only on x (horizontal dimension). Then it follows from the classical Terzaghi principle that $p_T(x) = p_s(s) + f(s)p_h$, where p_s is the structural pressure (different for compression and expansion), p_h is the hydrostatic pressure. The velocity v_s is obtained from the equation

$$\big(1 + \varphi_a/p_h\varphi_s q\big)\partial_z v_s = -\varphi_a/p_h\big(\partial_x p_T(x)/f(\varphi_s) - q(v_a - v_s)\partial_z\varphi_s\big) + \partial_z\big(\varphi_w(v_w - v_s) + \varphi_a(v_a - v_s)\big).$$

Then p_h and velocities v_w, v_a are expressed by explicit formulae from Darcy's law.

In the second model [3] the authors neglected the changes in the vertical dimension z and assumed that the gas phase is essentially at atmospheric pressure. They have used the mass continuity equation in the form $(d\varphi_w v_w)' = 0$. If $s = g(p_w)$ expresses the known relation between saturation and the capillary pressure, then using the Darcy law, we get the nonlinear convection-diffusion equation for unknown $p_w(x)$:

$$-\big(K_w(\varphi(x, d_{min}), g\big(p_w(x)\big))\big)' + \big(d(x, d_{min})\varphi(x, d_{min}), g(p_w(x))v_s\big)' = 0.$$

Unknown d_{min} can be found from the nonlinear equation, which follows from the Terzaghi principle. Then velocities can be found from Darcy's law.

Both models are based on the same two dimensional model and the same mechanical principles, but they are reduced to different 1D models. Our goal is to compare those two models.

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