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ON A FINITE DIFFERENCE SCHEME FOR BARATROPIC FILLING OF A VISCOUS GAS IN A LIMITED REGION

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Let us consider the initial-boundary problem describing a non-stationary baratropic filling of a viscous gas in a limited region

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0,$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] + \frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2} + \rho f, \qquad (1)$$

$$p = p(\rho).$$

The density ρ and the velocity u are functions of Euler variables

$$(t,x) \in Q = [0,T] \times [0,X].$$

Let the system (1) be complement with initial and boundary conditions

$$(\rho, u)|_{t=0} = (\rho_0, u_0),\ x \in [0,X],$$

$$\rho(t,0) = \rho_1(t),\ u(t,0) = u_1(t) > 0,\ u(t,X) = 0,\ t \in [0,T].$$

A finite difference scheme is constructed and investigated for the problem. Let $Q_{\tau,h}$ be a mesh in Q. An approximate solution in the nodes of $Q_{\tau,h}$ can be found from a system of linear algebraic equations which matrices are tridiagonal and nonsingular. An estimation of numerical integration error is proved in norm L_2 and equals $O(\tau + h^{3/2})$ under condition $\tau < C\sqrt{h}$.

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