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ADAPTATION OF CONVEX PROGRAMMING ALGORITHMS FOR SOLVING DISCRETE PROGRAMMING PROBLEMS

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In integer programming for some formulations it is possible to calculate the value of objective function only for integer-valued arguments. In this case for iteration methods does not exist the half-space of decreasing directions and the use of regular gradient methods is impossible.

But it is possible to adapt effective numerical methods of the convex programming for solving some classes of discrete programming problems.

The function $f\colon X \to R(X \subset R^n)$ is called discrete-convex if for all $x_i \in X(i = 1, \dots, n+1)$; $\lambda_i \geq 0$ $(i = 1, \dots, n+1)$ such as $\sum_{i=1}^{n+1} \lambda_i = 1$; $\sum_{i=1}^{n+1} \lambda_i x_i \in X$ holds $f(\sum_{i=1}^{n+1} \lambda_i x_i) \leq \sum_{i=1}^{n+1} \lambda_i f(x_i)$.

Remark 1. The use of all $n+1$ elements convex combinations follows from the well-known theorem of Caratheodory [1].

The graph of a discrete-convex function is a part of the graph of a convex function.

Lemma 2. *The function $f\colon X \to R(X \subset R^n)$ can be extended to convex function on $convX$ if f is discrete-convex on X . The convex extension of f_c is:*

$$f_c = \min\{\sum_{i=1}^{n+1} \lambda_i f(x_i) | x = \sum_{i=1}^{n+1} \lambda_i x_i; \sum_{i=1}^{n+1} \lambda_i = 1, \lambda_i \geq 0 (i = 1, \dots, n+1), x_i \in X (i = 1, \dots, n+1)\}$$

over x_i,$\lambda_i(i = 1, \dots, n+1)$.

Remark 3.

$$f_c(x) = \begin{array}{ll} \max\{\langle a, x\rangle | \langle a, x\rangle \leq f(x), x \in X\}, & x \ni X; \\ f(x), & x \in X \end{array}$$

The convex extension is so called point-wise maximum over all linear functions not exceeding the given function. The convex extension is a partly linear function.

Remark 4. The class of discrete functions is the largest one to be extended to the convex functions.

From point of view of integer programming $X = Z_+^n$ where $Z_+ = \{0, 1, 2, \dots\}$.

In this paper possibilities to construct iterations methods over points with integer coordinates converged to global optimum are considered. On each step of iteration the values of objective functions in some type of ends of n-dimensional unit cube are calculated until better value is founded. This better value is the next approximation. The conditions for convergence to global minimum are derived.

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REFERENCES

[1] R. Tyrrell Rockafellar. *Convex analysis.* Princeton University Press, New Jersey, 1970.