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INFINITE DETERMINANT METHOD IN STUDYING THE STABILITY OF HAMILTONIAN SYSTEMS WITH PERIODIC COEFFICIENTS

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We consider the hamiltonian system of differential equations

$$\frac{dx}{dt} = JH(t)x, \tag{1}$$

where $x^T = (x_1, x_2, \ldots, x_{2n})$ ia a $2n$-dimensional vector whose components x_k and x_{n+k} are the canonically conjugated variables, $J = \begin{pmatrix} 0 & E_n \\ -E_n & 0 \end{pmatrix}$ and E_n is the $n \times n$ identity matrix, $H(t)$ is the real-valued $2n \times 2n$ matrix function which can be represented in the form of the converging series

$$H(t) = H_0 + \varepsilon H_1(t) + \varepsilon^2 H_2(t) + \ldots, \tag{2}$$

where ε is a small parameter. The matrix functions $H_k(t)$ $(k = 1, 2, \ldots)$ in (2) are continuous and periodic with a period T, while H_0 is a constant matrix. Equations of the form (1) appear in many branches of science and engineering (see, for example, [1]). Our interest to the system (1) arises because such a system occurs in studying of the stability of equilibrium solutions in the elliptic restricted many-body problems [2].

In order to analyze the stability of the system (1) we have to calculate its characteristic exponents for $\varepsilon > 0$. Using the method of a small parameter we can find them in the form of power series in ε as it was done in [3], for example. But if we are looking for the stability boundaries the method of infinite determinant turns out to be more effective. It is based on the existence of periodic solutions on the boundaries between the domains of stability and instability. In the present paper we have developed the algorithm for analytical computation of the stability boundaries in the space of parameters and realized it for the second and the fourth order hamiltonian systems arising in the restricted many-body problems. The stability boundaries have been found in the form of powers series, accurate to the sixth order in a small parameter ε. All the computations are done with the computer algebra system *Mathematica*.

REFERENCES

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