Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

ON UNIQUENESS OF A SOLUTION TO NONLINEAR BOUNDARY VALUE PROBLEMS FOR TWO-DIMENSIONAL SYSTEM OF THE FIRST-ORDER DIFFERENTIAL EQUATIONS

VLADIMIR PONOMARJOV

Institute of Mathematics and Computer Science, University of Latvia

Raiņa bulv. 29, LV-1459, Rīga, Latvia

E-mail: felix@cclu.lv

Consider the system of two differential equations

$$x' = h(t, x, y), \quad y' = f(t, x, y) \tag{1}$$

together with the following boundary conditions

$$L_i(x(a), x(b), y(a), y(b)) = 0, \quad i = 1, 2 \tag{2}$$

where $h, f \in Car([a, b] \times R^2, R), \quad L_i \in C(R^4, R), \quad -\infty < a < b < +\infty$.

The problem (1), (2) was considered in [1] – [3].

By combining different one-sided Lipschitz conditions on the right sides h and f, the monotonicity conditions in the respective variables for the functions h, f, L_1, L_2, and independence of L_1, L_2, on some arguments, the total of 36 theorems can be proved. As a sample let us state the theorem.

Theorem 1. *Suppose that $h(t, x, y)$ is strictly increasing in y and $f(t, x, y)$ increases in x. Let L_1 be strictly increasing in the first variable and strictly decreasing in the third one and do not depend on the second and the fourth arguments. Let L_2 be increasing in all arguments and strictly increasing in the second one. Let also the conditions*

$$|h(t, x_1, y_1) - h(t, x_2, y_1)| \le K_1(t)|x_1 - x_2|,$$
$$f(t, x_1, y_1) - f(t, x_1, y_2) \ge K_2(t)(y_1 - y_2), \quad y_1 \ge y_2,$$

hold, where $K_1, K_2 \in L([a, b], [0, +\infty))$.

Then the boundary value problem (1), (2) has at most one solution.

REFERENCES

[1] V.V. Gudkov, Yu.A. Klokov, A.Ya. Lepin and V.V. Ponomarev. *Two-point boundary value problems for ordinary differential equations*. Zinatne, Riga, 1973.

[2] N.I. Vasilyev and Yu.A. Klokov. *Foundations of the theory of boundary value problems for ordinary differential equations*. Zinatne, Riga, 1978.

[3] V. Ponomarev About uniqueness of a solution of boundary value problems for a system of two first-order differential equations with linear boundary conditions, I. *The paper collection: "Mathematics. Differential equations." Univ. of Latvia, Institute of Math. and Comp. Sci.,* **4**, 2004, 73 – 80.