Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

NUMERICAL STUDY OF STABILITY OF A MAGNETIC-FLUID DROP ROTATING IN A NONUNIFORM MAGNETIC FIELD

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The axisymmetric magnetic-fluid drop of volume V situated on the rotating plane is considered. A permanent cylindrical magnet of radius R_0 and height $2b$ is located in line with the axis of rotation so that the fluid covers half a magnet surface. The problem on equilibrium shapes of the drop is solved in the dimensionless variables $r = R/R_0$, $z = Z/R_0$, $s = S/R_0$, $h = H/M_c$ where R and Z are the cylindrical coordinates, S is an arc length of an unknown meridian free-surface line, H is the magnetic-field intensity, M_c is the saturation magnetization for the cylinder. The problem for the parametric functions $r(s)$ and $z(s)$ is formulated by means of the general capillary-surface equations [1], and is as follows

$$r'' = z'(f + C), \quad z'' = -r'(f + C), \quad 0 < s < L; \quad f = \mathrm{Bo}_m \left(\mathrm{Fr}_m r^2 + h\right) + z'/r,$$
$$C = \frac{2}{r(L)} - \mathrm{Bo}_m \left(\frac{\mathrm{Fr}_m}{2} r^2(L) + \frac{2}{r^2(L)} \int_0^L r r' h \, ds \right); \quad 2\pi \int_0^L z r r' \, ds = \mathrm{U} + \pi\delta,$$
$$r(0) = z'(0) = r'(L) = z(L) = 0, \quad r'(0) = 1, \quad z'(L) = -1$$

where $\mathrm{Bo}_m = \mu_0 M_s M_c R_0/\sigma$ and $\mathrm{Fr}_m = \rho\omega^2 R_0^2/(2\mu_0 M_s M_c)$ are the magnetic Bond and Froude numbers, $\mathrm{U} = V/R_0^3$ is the dimensionless volume of the drop, $\delta = b/R_0$ is the relative height of the cylinder, μ_0 is the vacuum magnetic permeability, M_s is the saturation magnetization for the fluid, σ is the surface tension coefficient, ω is the angular velocity of the rotation. The magnetic intensity h is calculated analytically as the exact solution of the Maxwell's equations which is known in the case of a cylindrical magnet [2].

The problem is solved numerically by the iteration-difference method described in [1]. The method exactly responds to a physical instability of equilibrium free-surface shapes (see [1]) and makes it possible to evaluate a critical magnetic Froude number Fr_m^* corresponding to the onset of a crisis of equilibrium state under the action of centrifugal forces. Equilibrium shapes are determined at $\delta = 1$, $\mathrm{Bo}_m = 0$ and 10^4 over wide ranges of the parameters Fr_m and U up to their critical values. Dependence of the critical parameter Fr_m^* on the dimensionless drop volume U is established. Special attention is given to the theoretical justification of the critical values obtained.

REFERENCES

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