Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

INTEGRAL EQUATION FOR PARABOLIC PROBLEM WITH NONLOCAL BOUNDARY CONDITION

S. PEČIULYTĖ[1], M. SAPAGOVAS[2] and A. ŠTIKONAS[3]

[1,2,3] *Vytautas Magnus University,* [2,3] *Institute of Mathematics and Informatics*

[1,2,3] Vileikos g. 8, LT-44404, Kaunas, Lithuania, [2,3] Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: [1] `s.peciulyte@if.vdu.lt`, [2] `m.sapagovas@ktl.mii.lt`, [3] `ash@fm.vtu.lt`

Boundary problems with nonlocal conditions is a part of fast developing differential equations theory. In this research , there is analysed a modular parabolic problem with nonlocal boundary condition

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2} + f(x,t), \quad u(x,0) = \varphi(x), \quad u(0,t) = \gamma u(a,t) + d(t),\ a \in [0,1], \quad u(1,t) = \mu(t)$$

and classic first type initial-boundary problem

$$\frac{\partial w}{\partial t} = c\frac{\partial^2 w}{\partial x^2} + f(x,t), \quad w(x,0) = \varphi(x), \quad w(0,t) = z(t), \quad w(1,t) = \mu(t),$$

here $z(t) = u(0,t)$ is an unknown function. If this function is found, then while solving a classic problem a solution for the nonlocal boundary problem is also found. Solution of classic parabolic boundary problem can be expressed by Green function:

$$u(x,t) = \int_0^1 \varphi(\xi)G(x,\xi,t)d\xi + \int_0^t \int_0^1 f(\xi,\tau)G(x,\xi,t-\tau)d\xi d\tau + \\ +c\int_0^t z(\tau)H(x,t-\tau)d\tau - c\int_0^t \mu(\tau)H_1(x,t-\tau)d\tau,$$

here $H(x,t) = \frac{\partial}{\partial \xi}G(x,\xi,t)\big|_{\xi=0}$, $H_1(x,t) = \frac{\partial}{\partial \xi}G(x,\xi,t)\big|_{\xi=1}$. In this research, solution of the parabolic problem with nonlocal condition is searched by using classic problem solution phase. Then nonlocal boundary condition conduct into second type Voltera integral equation. Runge-Cutta type numerical algorithms can be used for solving integral equation within interval [0,T]. Moreover, parabolic problem can be analysed with other type of nonlocal conditions: when there is a derivation on the left or right boundary. Additionally, the problem can be analysed when solution's derivations are within both boundary conditions.

REFERENCES

[1] A.D. Poljanin. *Spravochnik po lineinym uravnenijam matematicheskoi fiziki.* Moskva: FML, 2001. (in Russian)

[2] B.M. Budak, A.A. Samarskii and A.N. Tihonov. *Sbornik zadach po matematicheskoi fizike.* Moskva: FML, 2003. (in Russian)

[3] M. Sapagovas and R. Čiegis. On some boundary problems with nonlocal conditions. *Differencialnye Uravnenija*, **23** (7), 1987, 1268 - 1274. (in Russian)

[4] M. Sapagovas. Hypothesis on the solvability of parabolic equations with nonlocal condition. *Nonlinear analysis: Modelling and control*, **7** (1), 2002, 93 - 104.