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# QUADRATIC AND CUBIC SPLINE COLLOCATION FOR VOLTERRA INTEGRAL EQUATIONS

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One of the most practical methods for solving Volterra integral equations of the second kind is the polynomial spline collocation method with step-by-step implementation. In the case of quadratic splines this method is stable only for the value $c = 1$ of the collocation parameter characterizing the position of collocation points between spline knots. It is also known that the cubic and higher order splines give unstable process for any choice of collocation parameter. We replaced an initial condition by a not-a-knot boundary condition at the other end of the interval. Our method is not any more step-by-step method, but it leads to system of equations which can be successfully done by Gaussian elimination. The main advantage which we get is the stability in the whole interval of collocation parameter for quadratic splines and in the interval $[1/2, 1]$ for cubic ones. Main results about stability and convergence are based on the uniform boundedness of corresponding spline interpolation projections. This is due to general convergence theorems for operator equations.

In the case $c = 1$ for quadratic splines and $c = 1/2$ for cubic ones the norms of collocation projections are of order $O(N)$ ($N$ being the number of used knots) and this allows to get convergence for smooth solutions. We have proved the regular convergence of operators in the case of quadratic splines which implied two-sided error estimates.

For quadratic splines it is also shown the convergence in the space of continuously differentiable functions. As well as in the space of continuous functions we get the uniform boundedness of collocation projections for all $c \in (0, 1)$ and the projection norms with linear growth for $c = 1$.

The numerical examples support these announced results.

## REFERENCES

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