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ESTIMATIONS OF THE NUMBER OF SOLUTIONS OF THE SECOND ORDER AUTONOMOUS BOUNDARY VALUE PROBLEMS

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We investigate the equations

$$x'' = -\alpha x + x^3, \quad \alpha > 0 \tag{1}$$

and

$$x'' = -\alpha x + x^2, \quad \alpha > 0 \tag{2}$$

together with the boundary conditions

$$x(0) = 0, \quad x(1) = 0 \tag{3}$$

and discuss generalizations of these problems.

The first equation has *the heteroclinic* solution (solution with "infinite period" [1, Ch. 1, § 1.4]) with the orbit connecting two saddle points at $(-\sqrt{\alpha}; 0)$ and $(\sqrt{\alpha}; 0)$.

Theorem 1. *The problem* (1), (3) *has exactly* $2i$ *nontrivial solutions if*

$$i^2\pi^2 < \alpha < (i+1)^2\pi^2, \quad i = 0, 1, \ldots \tag{4}$$

The orbits of this solution lie inside the region formed by two heteroclinic solutions connecting the saddle points.

The second equation has *the homoclinic* solution of infinite period [1] which starts and ends at the unique saddle point at $(\alpha; 0)$.

Theorem 2. *The problem* (2), (3) *has exactly* $2i + 1$ *nontrivial solutions if the condition* (4) *holds. The orbits of* $2i$ *solutions lie inside the region formed by the homoclinic solution, the orbit of one solution lies outside this region.*

REFERENCES

[1] R. Seydel. *Practical bifurcation and stability analysis. From equilibrium to chaos.* Springer, New York, 1994.