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A THIRD ORDER CORRECTION TO THE HELMHOLTZ EQUATION

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If a pure sine signal of high amplitude is sent to a bass loudspeaker with a reflex tube one will also hear a signal at twice the original frequency and higher harmonics. This non–linear distortion is due to the non–linear behaviour of the electromechanical driver, but also due to the non–linear transmission of the signal through the medium air. As non–linear effects play the essential role it is not enough to consider the equations of linear acoustics. On the other hand, solving the full time dependent Euler equations needs rather high effort, especially during the shape optimization process of the reflex tube.

Applying asymptotic analysis and assuming irrotational airflow one was able to derive a second order correction to the classical Helmholtz equation from the full Euler equations. This correction is again of Helmholtz type with non–homogeneous right hand side, which depends only on the first order solution, cf. [1]. In order to estimate the significance of the high order corrections at least the third order correction is needed. The same technique as in [1] to derive third order correction fails. The model in [1] was based on Lagrangian coordinates, hence the natural unknown function was the displacement function. For the first and second order corrections it was successfully shown, that the displacement function $\boldsymbol{\eta} = \boldsymbol{\eta}_1 + \varepsilon\boldsymbol{\eta}_2 + \ldots$ has a potential $\varphi = \varphi_1 + \varepsilon\varphi_2 + \ldots$ up to second order, i.e. $\boldsymbol{\eta}_1 = \nabla_{\boldsymbol{\xi}}\varphi_1$ and $\boldsymbol{\eta}_2 = \nabla_{\boldsymbol{\xi}}\varphi_2$, where ε is some small quantity and $\boldsymbol{\xi}$ is the Lagrangian spatial coordinate. We will show that the potential of the form $\varphi = \varphi_1 + \varepsilon\varphi_2 + \varepsilon^2\varphi_3 + \ldots$, generally, does not exist, i.e. $\boldsymbol{\eta}_3 \neq \nabla_{\boldsymbol{\xi}}\varphi_3$.

In [1] the author finally was interested in the behaviour of the pressure function. In our work we avoid the difficulties connected with the displacement function discussed above and get the third order correction for the pressure function, which is of Helmholtz type with non–homogeneous right hand side, which depends on the first and second order solutions. In order to complete the model, we prescribe suitable boundary conditions, which are rather complicated for the domains circumscribed in Lagrangian coordinates.

REFERENCES

[1] J. Mohring. Simulating bass loudspeakers requires nonlinear acoustics - a second order correction to the Helmholtz equation. In: *Progress in Industrial Mathematics at ECMI 2002*, Springer-Verlag, Heidelberg, 2002.