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SMALL PERTURBATIONS OF FREE INTERFACE DINAMICS FOR GAS BUBBLE IN THE MAGNETIC LIQUID ON ACCOUNT OF GRAVITATIONAL AND MAGNETIC FORCES

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Linear stability analysis for dynamics of bubble interface rising in vertical Hele-Shaw cell with magnetic liquid in perpendicular field is considered. Let $r = R[1 + \zeta(\alpha, t)]$ be the equation of the free interface for a bubble in polar coordinates (r, α) connected with its moving center, where $\zeta(\alpha, t)$ is a small dimenssionless perturbation for the circular bubble in the Hele-Shaw cells [1]. Linearizing for small perturbations $\zeta(\alpha, t)$ the Darcy equation and the corresponding boundary conditions, we obtain the following problem in the dimenssionless form:

$$\frac{\partial \zeta}{\partial t} = -Bg \sin\alpha \frac{\partial \zeta}{\partial \alpha} - 2Bg \cos\alpha.\zeta + \frac{\partial}{\partial r} p(r \exp(i\alpha))|_{r=1}, \tag{1}$$

$$p(\exp(i\alpha)) = -\frac{\partial^2 \zeta}{\partial \alpha^2} - (2Bg\cos\alpha + a)\zeta - \frac{Bm}{2h^2}\int_{-\pi}^{\pi}\Big[\frac{\zeta(\tau)}{\sqrt{\sin^2(\alpha-\tau)/2 + h^2/4}} - \frac{\zeta(\tau)-\zeta(\alpha)}{|\sin(\alpha-\tau)/2|}\Big]d\tau, \tag{2}$$

where h, Bg, Bm, a are constants, but the pressure $p(r \exp(i\alpha))$ is a harmonic function for $r > 1$. It satisfies special conditions when $r \to +\infty$: $p(r \exp(i\alpha) = c \ln r + O(1)$, where the constant c is determined by (1), (2) and by the equality $\int_{-\pi}^{\pi} \zeta(\tau)d\tau = 0$. In addition, initial values $\zeta|_{t=0} = \zeta_0(\alpha)$ also must be given here. Seeking the solution of this problem in the form $\zeta = \exp(\lambda t)u$ and $p(r \exp(i\alpha))|_{r=1} = \exp(\lambda t)P(\alpha)$ we obtain the spectral problem

$$\lambda u = -Bg \sin\alpha \frac{du}{d\alpha} - 2Bg\cos\alpha u + \frac{1}{4\pi} v.p. \int_{-\pi}^{\pi} \frac{P(\tau) - P(\alpha)}{\sin^2(\tau-\alpha)/2} d\tau, \tag{3}$$

$$P(\alpha) = -\frac{d^2u}{d\alpha^2} - (a + 2Bg\cos\alpha)u + \frac{Bm}{2h^2}\int_{-\pi}^{\pi}\Big[\frac{u(\tau)-u(\alpha)}{|\sin(\tau-\alpha)/2|} - \frac{u(\tau)}{\sqrt{\sin^2(\tau-\alpha)/2 + h^2/4}}\Big]d\tau \tag{4}$$

with periodic boundary conditions. This spectral problem is solved by DM methods analogously as it is done in [2] for Mathieu functions. They are based on the employment of Chebyshev polynomials. Positive eigenvalues and corresponding to them eigenforms have the greatest interest for applications.

REFERENCES

[1] A. Tsebers. Dynamics of magnetostatic instabilities. *Manetohydrodynamics*, **Vol.17** No.2, 1981, 113 – 121.

[2] T. Cirulis. Nonsaturated approximation by means of Lagrange interpolation. *Proceedings of the Latvian Academy of schiences. Section B*, **Vol.52** No.5(598), 1998, 234 – 244.