Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

HYPERBOLIC BY A PRESCRIBED VECTOR FIELD EQUATION AND BOUNDARY PROBLEMS

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We consider the linear differential equation with partial derivatives

$$\mathcal{L}(x, D)u = \sum_{|\alpha| \leq 2} a_\alpha(x) D^\alpha u = f(x), \tag{1}$$

where $a_\alpha(x)$, $f(x)$ are set in the limited region Q, n – dimensional Euclidean space $\mathbb{R}^n$ of the function of independent variables $x = (x_1, \ldots, x_n)$, D – is a vector of differentiation $D = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}\right)$, $D^\alpha = D_1^{\alpha_1} \ldots D_n^{\alpha_n} = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \ldots \partial x_n^{\alpha_n}}$, $\alpha = (\alpha_1, \ldots, \alpha_n)$ – multiindex, $|\alpha| = \alpha_1 + \ldots + \alpha_n$.

We will introduce the definition of the linear hyperbolic equation with respect to the set vector field. Suppose that vector field $\mathcal{N}$ of the class C^1, is defined in $\mathbb{R}^n$ and consists of elements-unit vectors $\eta(x) = (\eta_1(x), \ldots, \eta_n(x))$, $|\eta|^2 = \eta_1^2 + \ldots + \eta_n^2 = 1$.

Definition 1. Equation (1) in point x with respect to the direction $\eta(x)$ will be referred to as hyperbolic if

(i) polynomial $\mathcal{L}_0(x, \eta(x)) = \sum_{|\alpha|=2} a_\alpha(x) \eta_1^{\alpha_1}(x) \ldots \eta_n^{\alpha_n}(x) \neq 0$, for definiteness we assume $\mathcal{L}_0(x, \eta) \geq \delta$ and δ – some positive integer;

(ii) polynomial $\mathcal{L}_0\left(x, \tau\eta(x) + \xi(x)\right)$ with respect to $\tau \in \mathbb{R}^1$ has two real different roots where $\xi(x) = (\xi_1(x), \ldots, \xi_n(x))$, $|\xi(x)| = 1$, $(\eta(x), \xi(x)) = \sum_{k=1}^{n} \eta_k(x)\xi_k(x) = 0$.

Equation (1) is hyperbolic in closure $\overline{Q} \subset \mathbb{R}^n$ of region Q, if it is hyperbolic in each point $x \in \overline{Q}$ with respect to the direction $\eta(x)$ from the $\mathcal{N}$ set in $\overline{Q}$ vector field.

A theorem of the existence and uniqueness of a strong solution has been proved for boundary problems of such equation (1). Mollifiers with variable step in the theory of statements of correct problems are used as well as proofs of their solubility for partial differential equations. The existence and uniqueness of strong solutions for boundary problems of equation (1) in appropriate functional spaces is proved by the method of energy inequality and mollifiers with variable step.