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THE TOTAL ERROR ESTIMATE OF ONE REGULARIZATION METHOD FOR THE SOLUTION OF FIRST KIND OPERATOR EQUATIONS WITH INEXACTS BOTH OPERATORS AND RIGHT-HAND SIDES

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In this paper we consider an operator equation of the first kind

$$Az = u, \tag{1}$$

where H is a Hilbert space, the operator $A: H \to H$ is linear, self-conjugate, positive and continuous, $u \in AH$ is a given element and $z \in H$ is the unknown element.

In this paper it is assumed that the equation (1) is solvable for all $u \in H$, i.e. $AH = H$. Let it is known a'priory information that the equation (1) has the exact solution z^p under the given exactly initial data $\{A^p; u^p\}$ where $A = A^p: H \to H$ is the exact operator, and $u = u^p \in H$ is the exact right-hand side of the equation (1), i.e. $A^p z^p = u^p$. In the work [1] is considered the method for construction of the approximate solution of the equation (1) which is stable against the small changes of the initial information when only right side of the equation (1) was inexact, but the operator was assumed exactly known, i.e. instead of $u = u^p$ there was $\{u^\delta; \delta\}$ such that $\|u^p - u^\delta\| \le \delta$.

In this paper, in contrast to [1], we will consider a case when instead of exacts initial data $\{A^p; u^p\}$ there are approximately initial data $\left\{A^h, h \,;\, u^\delta, \delta\right\}$. When we have such information about the equation (1) then we could find only approximate solution of the equation (1). Besides this approximate solution converges to the exact solution z^p as δ and h converge to zero independently.

Such problem is considered in the works [2; 3; 5]. In these papers are proved the existence theorems of regularizing operators obtaining the way of variation methods: is minimized Tichonoff stabilization functional $\Omega\,[z]$. In the works [4; 6; 7; 8; 9] are investigated the various questions relating to similar problem.

In the present work author considers the concrete iterative method for the solution of such first kind operator equations in the abstract Hilbert space and we estimate its total degree of convergence without any additional conditions.

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