Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

THE TAYLOR SERIES EXPANSION COEFFICIENTS OF SOLUTIONS OF THE EMDEN - FOWLER TYPE EQUATIONS

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We present the explicit non-recursive formulae for the Taylor series expansion coefficients of the functions $S_n(t)$ defined as a solution of the Emden - Fowler equation $x'' = -nx^{2n-1}$ with the initial conditions $x(0) = 0$, $x'(0) = 1$, where $n = 1, 2, \ldots$ Using the Ostrowski - Zabreiko - Lysenko formula [1], [2] one obtains

Theorem. *The nontrivial coefficients of the Taylor series expansion for the function $S_n(t)$ at $t = 0$ are given by $s_1 = 1$ and*

$$s_{2nk+1} = \frac{1}{2nk+1} \sum_{\beta_1+2\beta_2+\cdots+k\beta_k=k} (-2)^{-2k+\sum\limits_{i=1}^{k}\beta_i} (V;\beta_1,\ldots,\beta_k,2nk) \cdot \frac{\prod\limits_{i=1}^{k} \binom{2i-1}{i-1}^{\beta_i}}{\prod\limits_{i=1}^{k} (2ni+1)^{\beta_i}}, \tag{1}$$

where $(V;\beta_1,\ldots,\beta_k,2nk) = \dfrac{(\beta_1+\beta_2+\cdots+\beta_k+2nk)!}{\beta_1!\beta_2!\cdots\beta_k!(2nk)!}$, $k = 1, 2, \ldots$

Corollary 1. The function $S_1(t)$ coincides with the elementary sine $\sin(t)$ with the well known nontrivial coefficients of the Taylor series expansion at $t = 0$: $s_{2k+1} = \frac{(-1)^k}{(2k+1)!}$. Using (1) one obtains the set of identities:

$$\sum_{\beta_1+2\beta_2+\cdots+k\beta_k=k} (-2)^{-2k+\sum\limits_{i=1}^{k}\beta_i} (V;\beta_1,\ldots,\beta_k,2k) \cdot \frac{\prod\limits_{i=1}^{k} \binom{2i-1}{i-1}^{\beta_i}}{\prod\limits_{i=1}^{k} (2i+1)^{\beta_i}} = \frac{(-1)^k}{(2k)!} \quad (k = 1, 2, \ldots).$$

Corollary 2. The function $S_2(t)$ coincides with the lemniscatic sine $\mathrm{sl}(t)$ [3], which can be expressed through the Jacobian elliptic functions. Hence (1) gives the nontrivial coefficients of the Taylor series expansion at $t = 0$ for the lemniscatic sine $\mathrm{sl}(t) = \mathrm{sn}(t; i)$.

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