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ON THE POPULATION MODEL WITH A SINE FUNCTION

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The exponential models have limited predictive power in population problems since as time passes the predicted population becomes so large that it is no longer realistic. For most biological species is valid the population increases until it reaches a certain upper limit. Then, due to the limitations of available resources, the creatures will become testy and engage in competition for those limited resources. In 1845 Pierre-Francois Verhulst (1804-1849) had to offer for investigation of population the model

$$x(n+1) = \mu x(n)(1 - x(n)),$$

where $x(n)$ be the size of a population at time n and μ is the rate of growth of the population. This equation is the simplest nonlinear first-order difference equation but it has complicated dynamics. The quadratic function $h(x) = \mu x(1-x)$, $x \in [0,1]$, is called also *logistic function*. The logistic function is widely studied (see, for example, [2; 3; 4]).

Similar behavior in the interval $[0,1]$ is also for function

$$s_a(x) = a \sin \pi x$$

($a > 0$ - parameter). If we fix x_0 and consider the orbit $\{x_0, s_a(x_0), s_a(s_a(x_0)), \ldots\}$ then as for logistic function the dynamic of $s_a(x)$ is very complicated.

We prove that for every $a > \frac{\sqrt{\pi^2+1}}{\pi}$ exist subset $\Lambda_a \in [0,1]$ such that

1. $s_a \colon \Lambda_a \to \Lambda_a$,
2. the periodic points of s_a are dense in Λ_a,
3. s_a is topologically transitive in Λ_a,
4. s_a exhibits sensitive dependence on initial conditions in Λ_a, i.e. $s_a \colon \Lambda_a \to \Lambda_a$ is chaotic function by R. Devaney [1].

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