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ON THE APPROXIMATION OF A PROBABILITY DENSITY FUNCTION BY SMOOTHING SPLINES

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The histopolation problem for a given histogram $F = \{f_1, \dots, f_n\}$ on a mesh $\triangle_n$: $a = t_0 < t_1 < \dots < t_n = b$ by a function g from Sobolev space $W_2^r[a,b]$ is formulated as the minimization problem

$$\int_a^b (g^{(r)}(t))^2\,dt \longrightarrow \min_{g\in W_2^r[a,b],\quad \int_{t_{i-1}}^{t_i} g(t)\,dt = f_i h_i,\quad i=1,\dots,n}, \tag{1}$$

where $h_i = t_i - t_{i-1}$, $i \in I$, $I = \{1, \dots, n\}$.

We consider the problem of approximation of histogram F under inequality constraints for area matching conditions

$$\int_a^b (g^{(r)}(t))^2\ dt \longrightarrow \min_{|\int_{t_{i-1}}^{t_i} g(t)\,dt - f_i h_i| \le \varepsilon_i,\quad \varepsilon_i \ge 0,\quad i\in I,\quad \int_a^b g(t)\,dt = 1} \tag{2}$$

We investigate its solution which is a spline s of one variable from the space S of integral splines of degree $2r$ and defect 1 over the mesh $\triangle_n$

$$S = \left\{ s \in W_2^r[a,b] :\ \forall g \in W_2^r[a,b] \int_{t_{i-1}}^{t_i} g(t)\,dt = 0,\ i \in I,\ \Longrightarrow \int_a^b g^{(r)}(t) s^{(r)}(t)\,dt = 0 \right\}.$$

For construction of this solution the modification of the method described in [1] for the solution of problem (2) considered without condition $\int_a^b g(t)\,dt = 1$ is suggested.

REFERENCES

[1] N. Budkina. On a method of construction of smoothing histosplines. *Proc. Estonian Acad. Sci. Phys. Math.*, to appear.