Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

ON MODELING OF TEMPERATURE DISTRIBUTION WITHIN A THIN MATERIAL SHEET UNDER CONDUCTIVE-RADIATIVE HEAT TRANSFER

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In the paper [2] are considered mathematical models for description of conductive-radiative heat transfer. As a result, complicated integro-differential boundary value problems are derived that model heat propagation within a physical system.

Let us consider a simplified situation from [1]. There a glass fabric sheet is heated up in a furnace. As conductive-radiative heat transfer occurs in the furnace-fabric sheet system, the resulting temperature distribution T in the glass fabric sheet Ω can be found after solving of the following elliptic boundary value problem:

$$k_1 \Delta T - k_2 \frac{\partial T}{\partial x_1} = 0 \quad \text{in } \Omega,$$

$$-k_1 \frac{\partial T}{\partial n} = G_1(|T|^3 T) - G_2(|T_{ht}|^3 T_{ht}) + k_3(T - T_g) \quad \text{on } \Sigma_s,$$

$$T = T_0 \quad \text{on } \partial\Omega \backslash \Sigma_s.$$

Here linear operators $G_1\colon L_{5/4}(\Sigma_s) \mapsto L_{5/4}(\Sigma_s)$, $G_2\colon L_{5/4}(\Sigma_{ht}) \mapsto L_{5/4}(\Sigma_s)$ that are given in the implicit form describe radiative heat propagation within Σ_s, Σ_{ht} system, where Σ_s is a lateral surface of Ω, Σ_{ht} is an active surface within the furnace, that can emit and reflect radiation. T_{ht} is temperature distribution on Σ_{ht}, whereas T_g is temperature of the surrounding air.

Unfortunately, it is very hard to carry out any numerical calculations for this boundary value problem with standard numerical methods. The main reason is the fact that the sheet geometry is strongly degenerated in this situation. Thickness-width ratio for the sheet can achieve the value 1/15000.

To find a workaround, we analyzed the dependence of T from the thickness ϵ of Ω and found that averaged (in thickness direction) temperature $\tilde{T}$ in the limit, as $\epsilon \to 0$, satisfies a nonlinear equation:

$$\tilde{G}_1(|\tilde{T}|^3 \tilde{T}) + k_3 \tilde{T} = \tilde{G}_2(|T_f|^3 T_f) + k_3 \tilde{T}_g,$$

where $\tilde{G}_1$ and $\tilde{G}_2$ are explicitly defined by G_1 and G_2.

REFERENCES

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